Inviscid oscillations of bubbles and drops

+ Oscillations of a bubble in surfactant solutions

+ Rebound of a bubble from a solid in surfactants

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Motivation

Bubble rebound from a solid surface is suppressed, if a surfactant is present.

Left:
1 mm bubble
pure water
σ = 72 mN/m

Right:
1 mm bubble
water + surfactant
σ = 62 mN/m

ADSA: "used surfactant does not seem to be a surfactant"

Bubble pressure tensiometry: Some dependence of the surface tension on the age

How to predict this and similar processes?

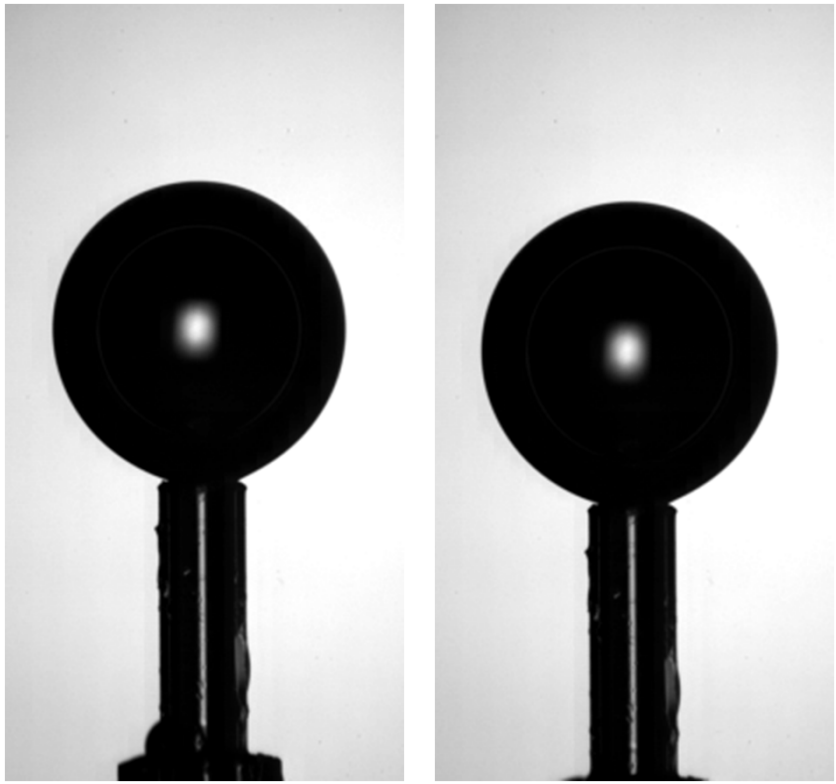
Motivation

Shape oscillations are strongly affected by presence of surfactants.

Possible enlargement of drop-shape-analysis (ADSA) methods.

Oscillations of attached bubble (*D* = 1.2 mm)

in pure water | in surfactant solution



Outline

1. shape oscillations of an attached bubble
   - **inviscid analysis**
   - **in pure liquids**, no gravity effects

2. shape oscillations of attached bubble
   - **experiments**
   - in pure liquid, **with gravity effects**

3. shape oscillations of attached bubble
   - experiments
   - **in surfactant solutions**, with gravity effects

4. **bubble rebound from solids**
   - experiments, in surfactant solutions
   - **correlation of restitution coefficient**

5. **shape oscillations of freely-floating bubbles**
   - experiments
   - in surfactant solutions

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Inviscid analyses

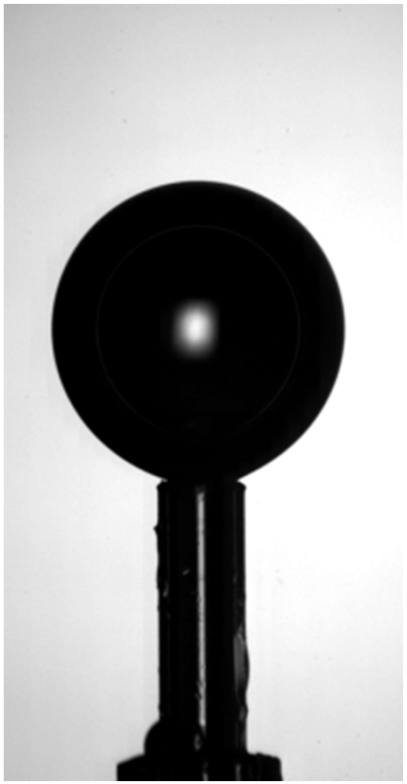
Experiments:

Results in surfactant solution are compared with those obtained in pure liquid.

The difference is considered as a consequence of surfactant effects.

? **Can we predict the case of pure liquid without doing experiments?**

pure liquid
reference case



surfactant solution
tested case

Inviscid analyses

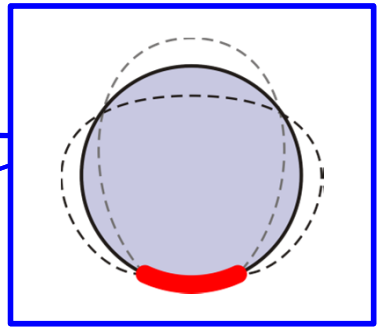
Available analyses:

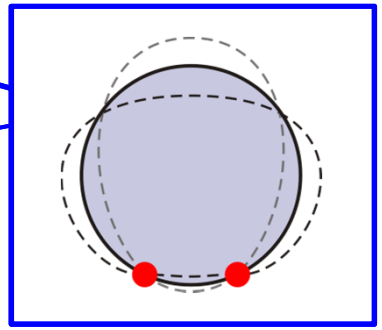
Strani & Sabetta (JFM 1984)
(inviscid)

Strani & Sabetta (JFM 1988)
(viscous)

Bostwick & Steen (PoF 2009)
(inviscid)

Prosperetti (PoF 2012)
(inviscid)



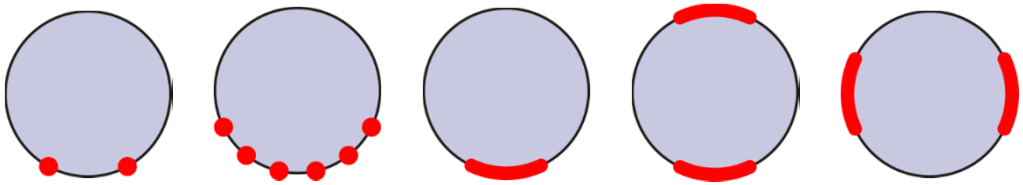


Present analysis:

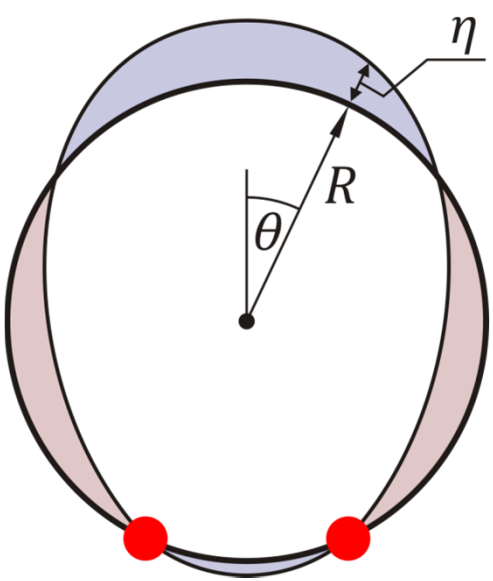
- inviscid (irrotational flow)
- linear (small amplitudes)
- no gravity (spherical shape)

Advantages:

- straightforward and convenient
- adapts easily to any constrain
- **response to support motion or to volume variations**



Inviscid analysis – shape description



Deviation from a sphere:

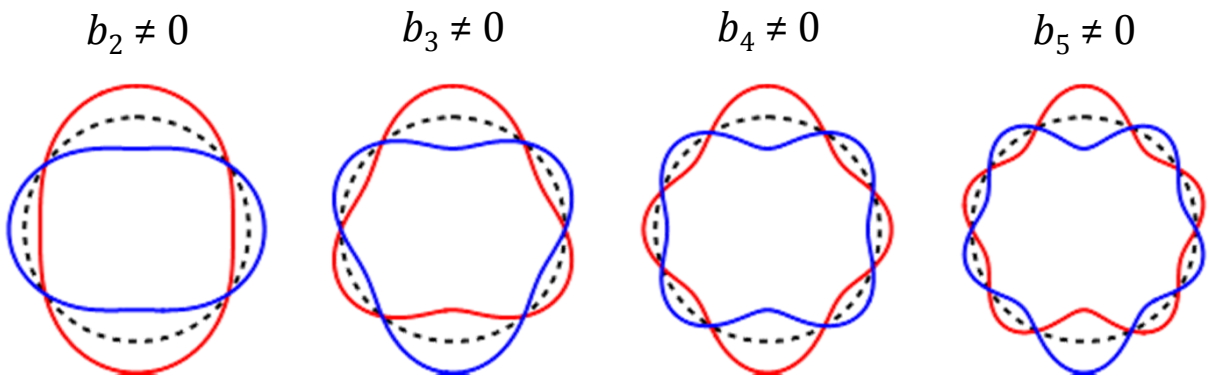
$$\eta(t,\theta)$$

This deformation is decomposed in series

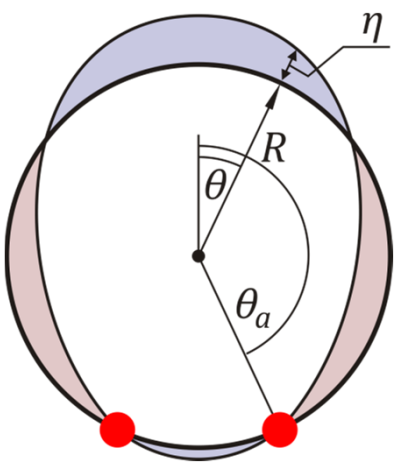
$$\eta(t,\theta) = \boxed{b_0(t)}P_0(\cos\theta) + \boxed{b_1(t)}P_1(\cos\theta) + \boxed{b_2(t)}P_2(\cos\theta) + \ldots$$

(P_k's are Legendre polynomials)

→ **the shape is described by $\boxed{b_j}$ coefficients**



Inviscid analysis - constraints



The deformation is decomposed in series

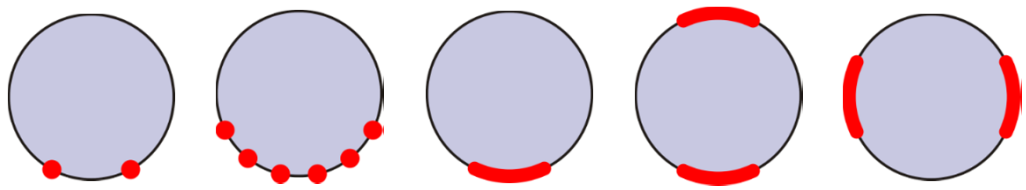
$$\eta(t,\theta) = \boxed{b_0(t)}P_0(\cos\theta) + \boxed{b_1(t)}P_1(\cos\theta) + \boxed{b_2(t)}P_2(\cos\theta) + \ldots$$

Because of attachment, $\boxed{b_j}$'s are not independent.

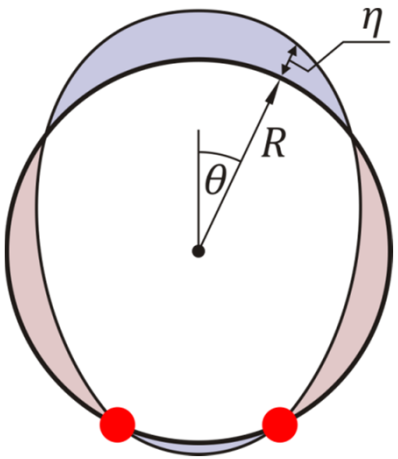
Constraints:

$$g(\boxed{b_1, b_2, \ldots, b_N}, \boxed{t}) \equiv \sum_{j=1}^{N} \boxed{b_j} P_j(\cos\theta_a) - \begin{matrix}\text{prescribed}\\ \text{motion}\boxed{(t)}\end{matrix} = 0$$

All these constrains can be expressed in a similar way:



Inviscid analysis - energies



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For this decomposition

- **potential (surface) energy** is

$$V = \sigma\Delta S = 2\pi\sigma\sum_{j=1}^{N}\frac{j^2+j-2}{2j+1}\boxed{b_j^2}$$

- irrotational velocity field is assumed (it is known)
- **kinetic energy** of the flow is

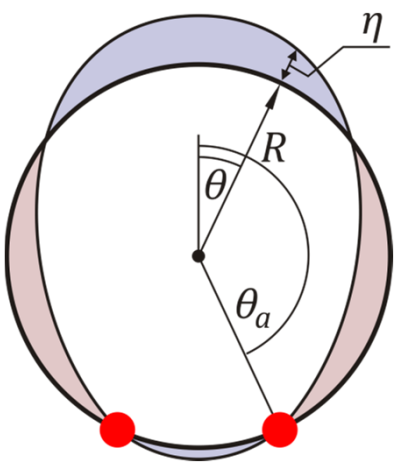
$$T = 2\pi R^3\sum_{j=1}^{N}\left(\frac{\rho_i}{j(2j+1)}+\frac{\rho_e}{(j+1)(2j+1)}\right)\boxed{\dot{b}_j^2}$$

- **energy dissipation rate** is

$$D = 4\pi R\sum_{j=1}^{N}\left(\mu_i\frac{j-1}{j}+\mu_e\frac{j+2}{j+1}\right)\boxed{\dot{b}_j^2}$$

→ system of ODE's for $\boxed{b_j}$ coefficients can be obtained using variational methods

Inviscid analysis – ODEs for shape



The deformation is decomposed in series

$$\eta(t,\theta) = b_0(t)P_0(\cos\theta) + b_1(t)P_1(\cos\theta) + b_2(t)P_2(\cos\theta) + \ldots$$

→ Equations for b_j can be obtained via **Lagrange's equations**

→ constraints taken in account by method of **Lagrange λ-multipliers**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{b}_j}\right) - \frac{\partial T}{\partial b_j} + \frac{\partial D}{\partial \dot{b}_j} + \frac{\partial V}{\partial b_j} = \sum_{k=1}^{M} \lambda_k \frac{\partial g_k}{\partial b_j}$$

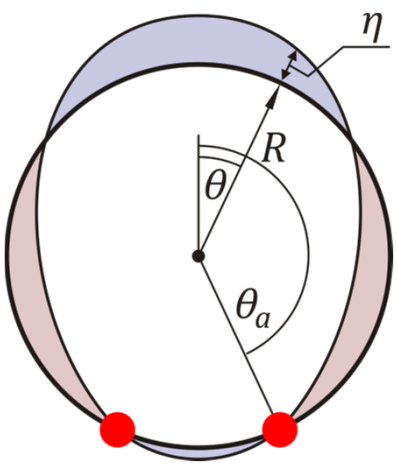
Resulting equations:

$$\left(\frac{\rho_i}{\rho_0}\frac{1}{j(2j+1)} + \frac{\rho_e}{\rho_0}\frac{1}{(j+1)(2j+1)}\right)\ddot{b}_j + 2\frac{\mu_0}{\rho_0 R^2}\left(\frac{\mu_i}{\mu_0}\frac{j-1}{j} + \frac{\mu_e}{\mu_0}\frac{j+2}{j+1}\right)\dot{b}_j +$$

$$+ \frac{\sigma}{\rho_0 R^3}\left(\frac{j^2+j-2}{2j+1}\right)b_j - \frac{1}{4\pi\rho_0 R^3}\sum_{k=1}^{M}\lambda_k P_j(\cos\theta_{a,k}) = 0$$

$$\sum_{j=1}^{N} b_j P_j(\cos\theta_a) - \text{prescribed motion}(t) = 0$$

Inviscid analysis – ODEs for shape



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Resulting equations (matrix form):

$$\mathbf{M}\frac{\mathrm{d}^2\mathbf{b}}{\mathrm{d}t^2} + 2\frac{\mu_0}{\rho_0 R^2}\mathbf{D}\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}t} + \frac{\sigma}{\rho_0 R^3}\mathbf{K}\mathbf{b} = \mathbf{f}$$

vector of b_j and λ_k

"mass" matrix (diagonal)

"damping" matrix (diagonal)

"rigidity" matrix (sparse)

forcing vector (non-zero, if support moves or volume changes)

Inviscid analysis – summary

Analysis – main features:

Bubble shape is decomposed in

$$\eta(t,\theta) = \boxed{b_0(t)}P_0(\cos\theta) + \boxed{b_1(t)}P_1(\cos\theta) + \boxed{b_2(t)}P_2(\cos\theta) + \ldots$$

Ordinary differential equations for $\boxed{b_j}$ coefficients are then obtained by computing energies and by application of Lagrange's equation and Lagrange multipliers.

Advantages:

- analysis easily adapts to any constraints
- analysis allows computing response to
  - support motion
  - volume variations

Drawbacks:

- no gravity effects (only spherical basic shape)
- inviscid = cannot handle surfactants

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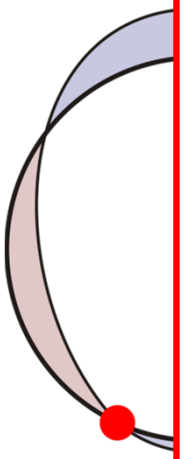

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) + …

tipliers

"mass"
(diag

ort
nges)

"damping" matrix
(diagonal)

(sparse)

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   - **correlation of rebound coefficient**

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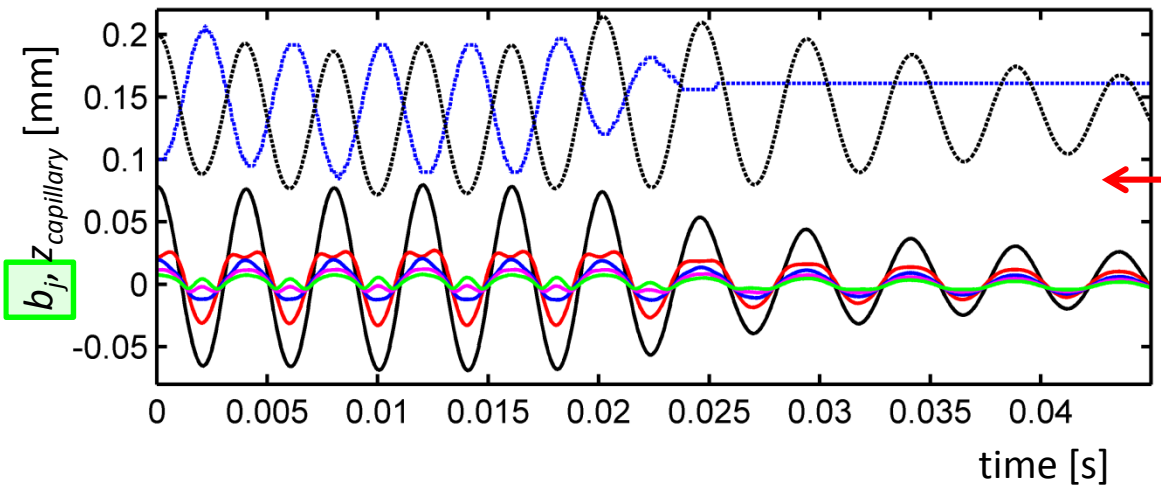
Pure liquid, experimental protocol

Decomposition of the experimental shape:

$$\eta(t,\theta) = \boxed{b_0(t)}P_0(\cos\theta) + \boxed{b_1(t)}P_1(\cos\theta) + \boxed{b_2(t)}P_2(\cos\theta) + \ldots$$

… evolution of $\boxed{b_j}$ coefficients is obtained experimentally…

bubble (0.93 mm) at 250 µm capillary
forced at 250 Hz



mode 1

Pure liquid, experimental protocol

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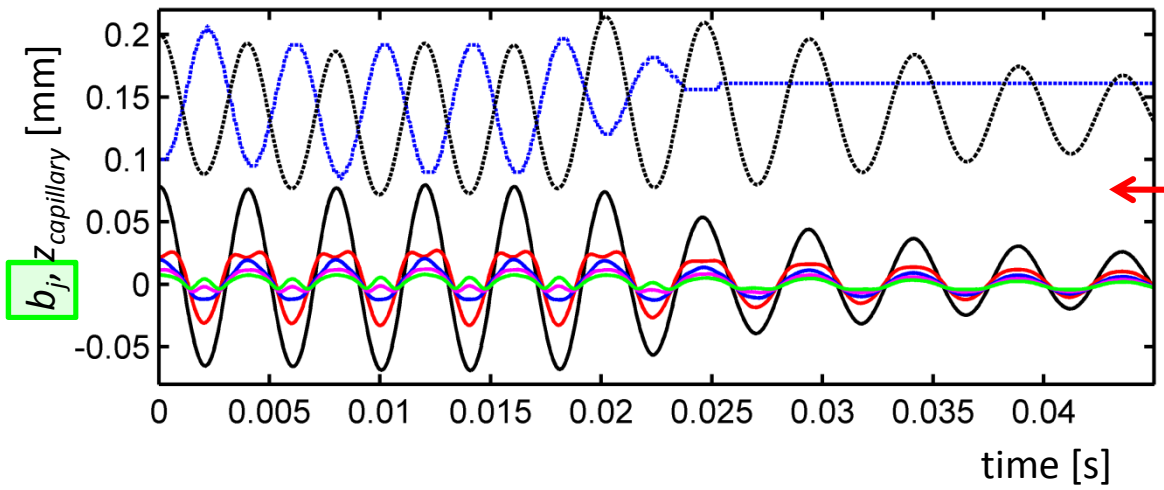
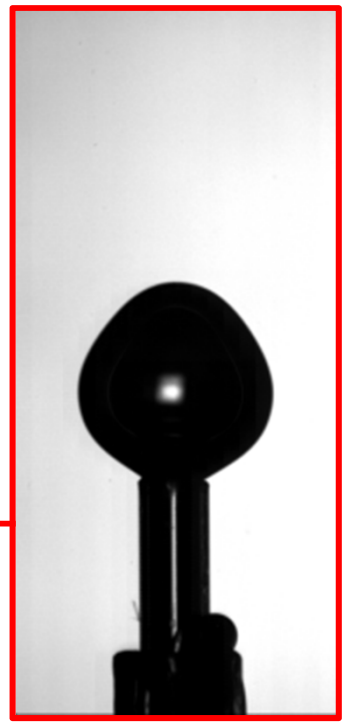
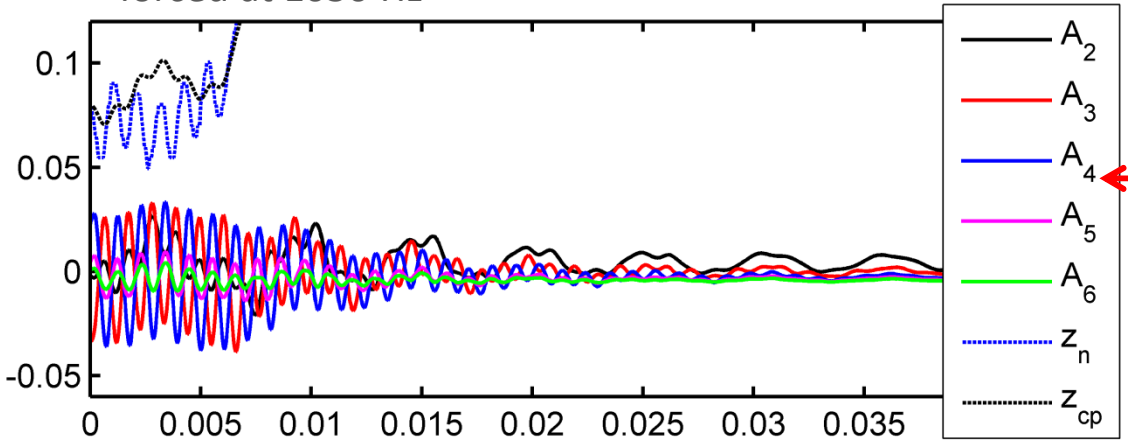


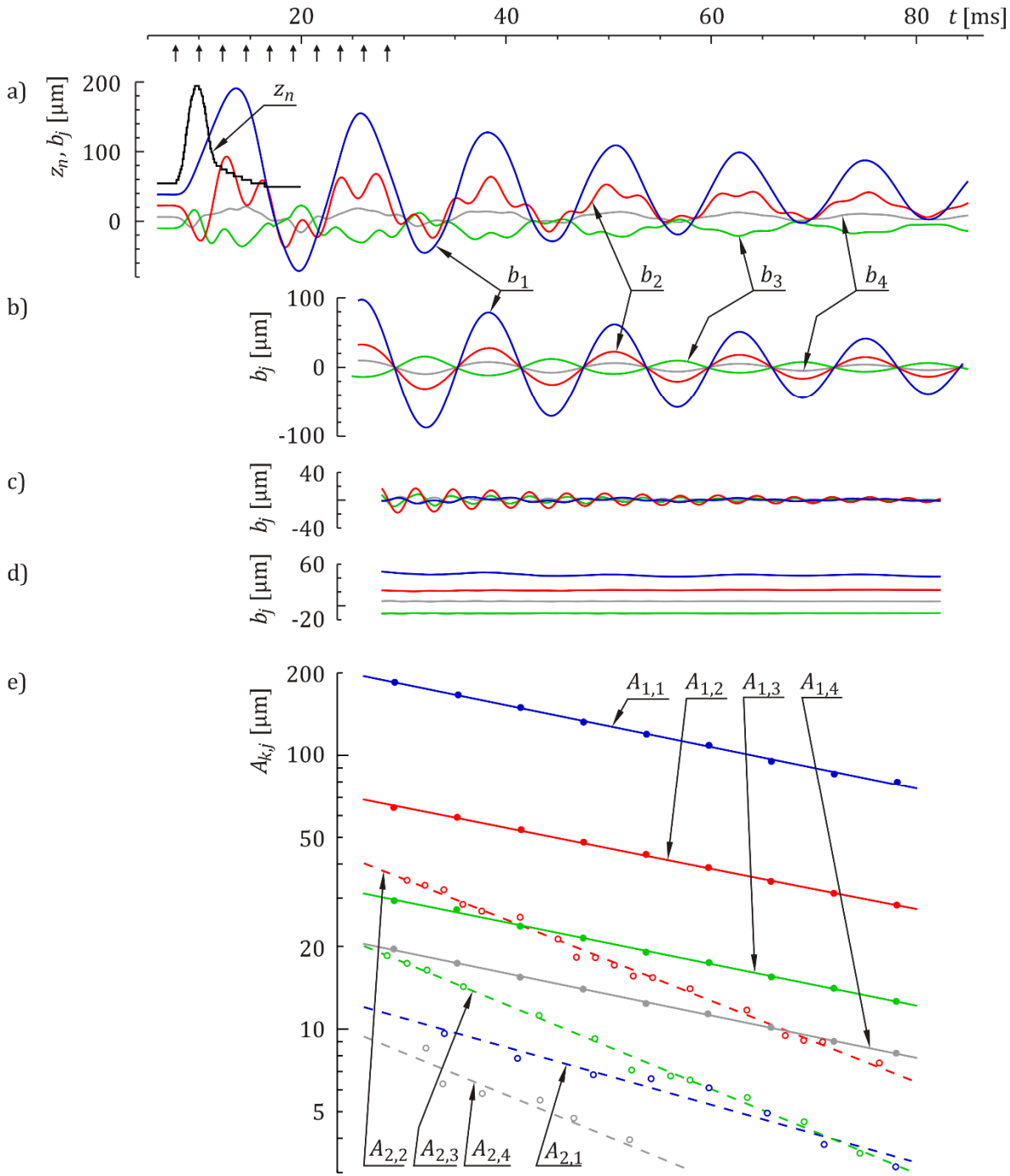
mode 1

mode 4



forced at 1050 Hz

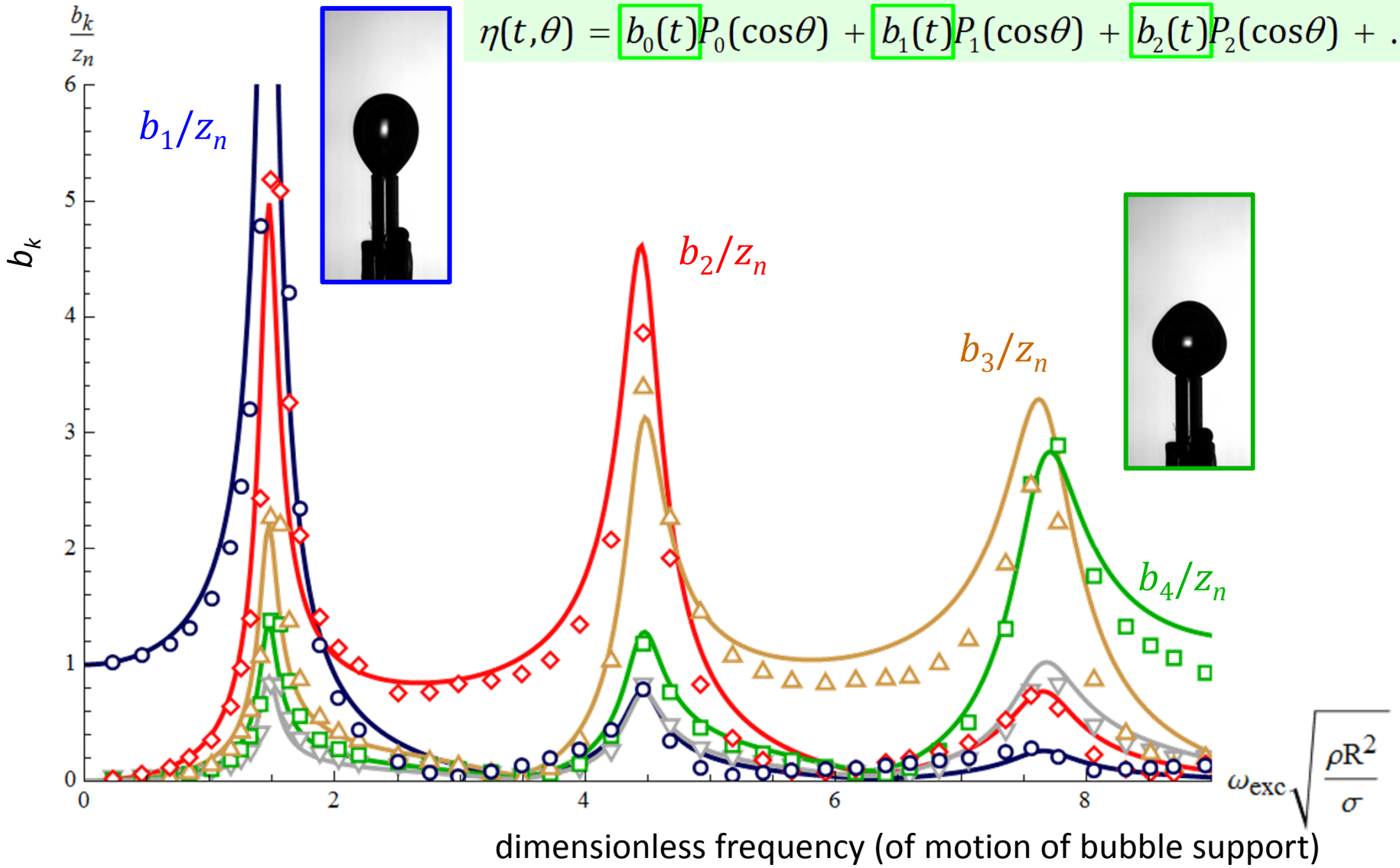



20
40
60
80
t [ms]
a)
200
100
0
z_n, b_j [µm]
z_n
b_1
b_2
b_3
b_4
b)
100
0
-100
b_j [µm]
c)
40
-40
b_j [µm]
d)
60
-20
b_j [µm]
e)
200
100
50
20
10
5
A_{kj} [µm]
$A_{1,1}$
$A_{1,2}$
$A_{1,3}$
$A_{1,4}$
$A_{2,2}$
$A_{2,3}$
$A_{2,4}$
$A_{2,1}$

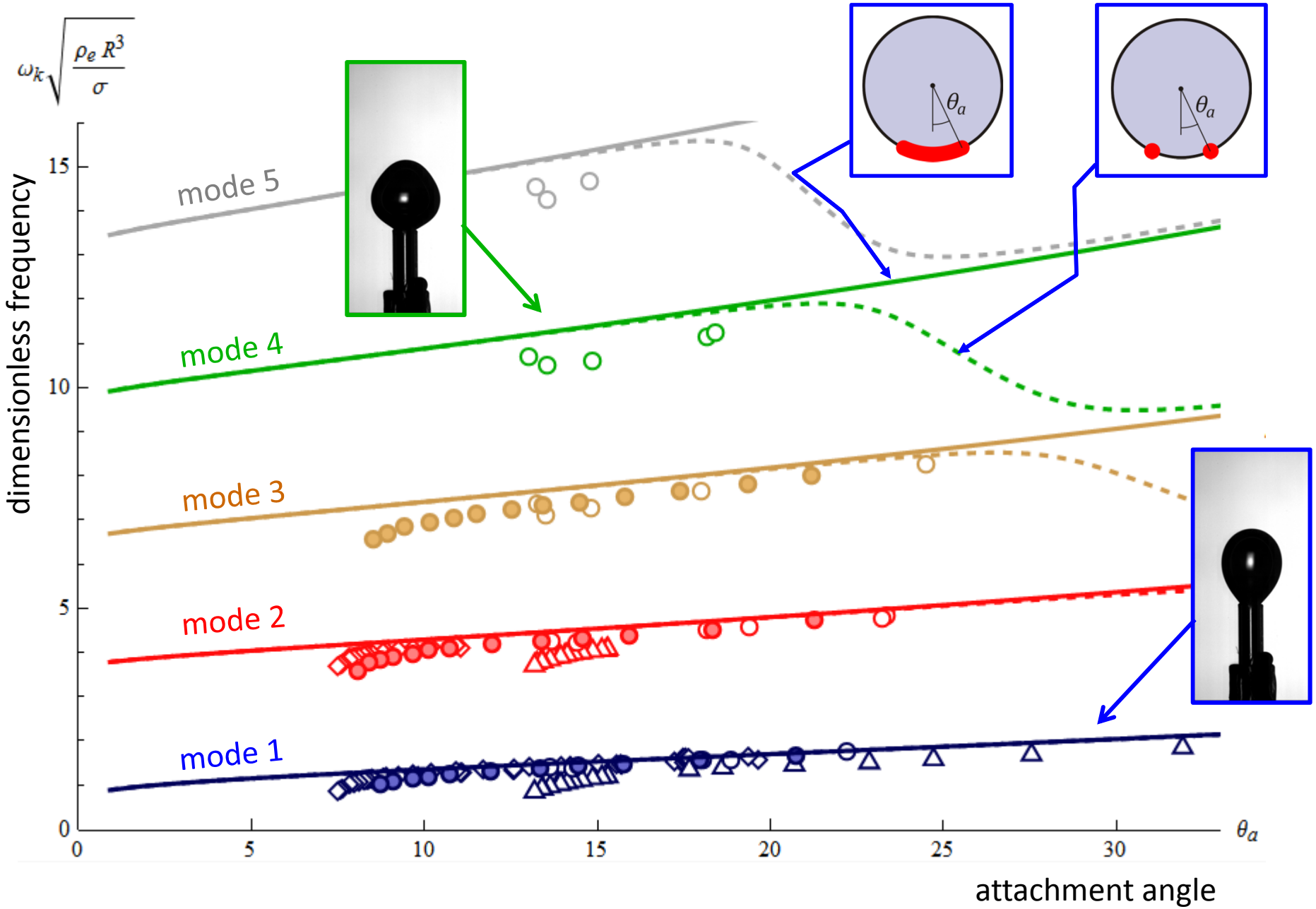
Pure liquid: frequency transfer function

(response to motion of bubble support)

$$\eta(t,\theta) = \boxed{b_0(t)}P_0(\cos\theta) + \boxed{b_1(t)}P_1(\cos\theta) + \boxed{b_2(t)}P_2(\cos\theta) + \ldots$$



Pure liquid: eigenmode frequency



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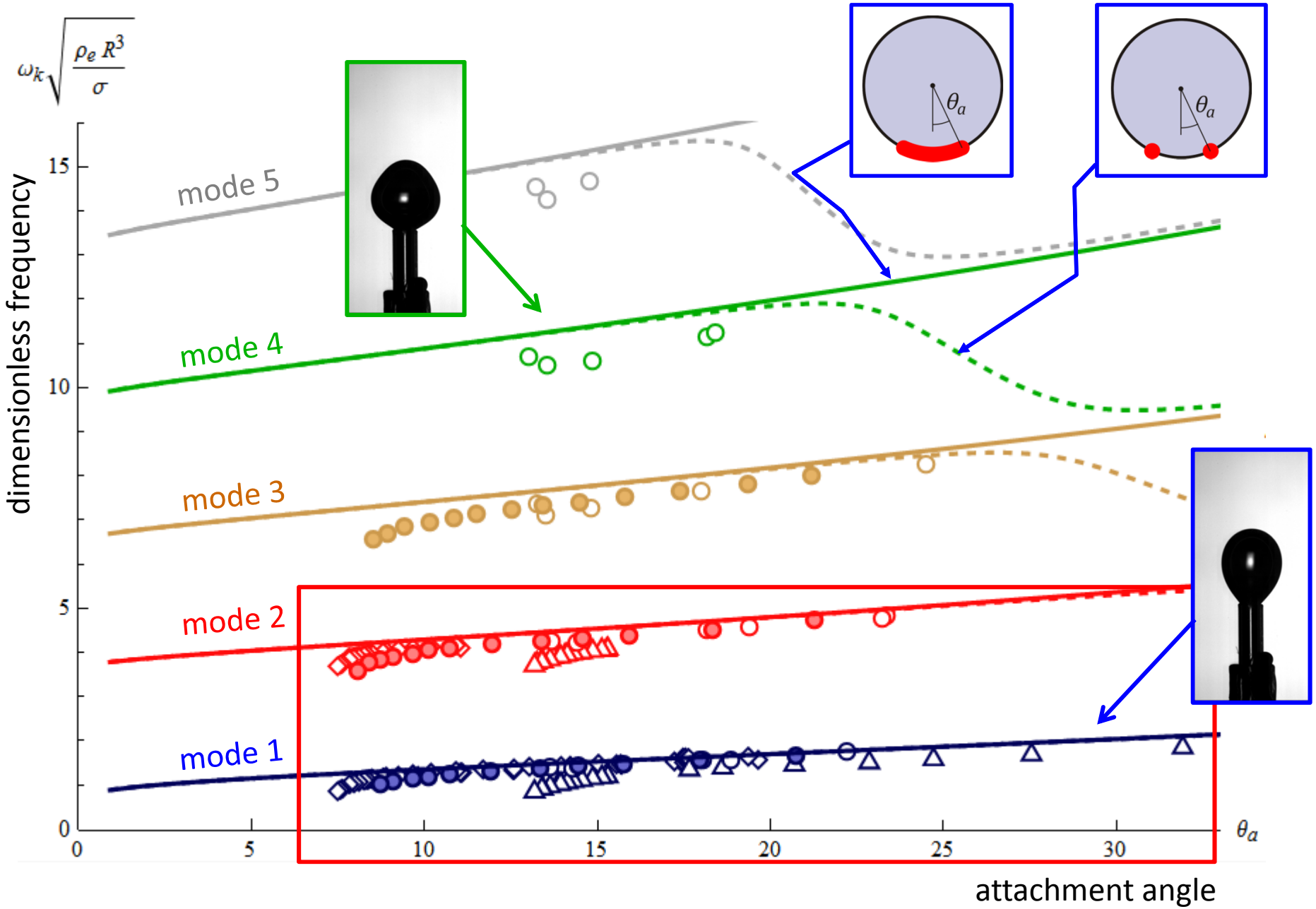
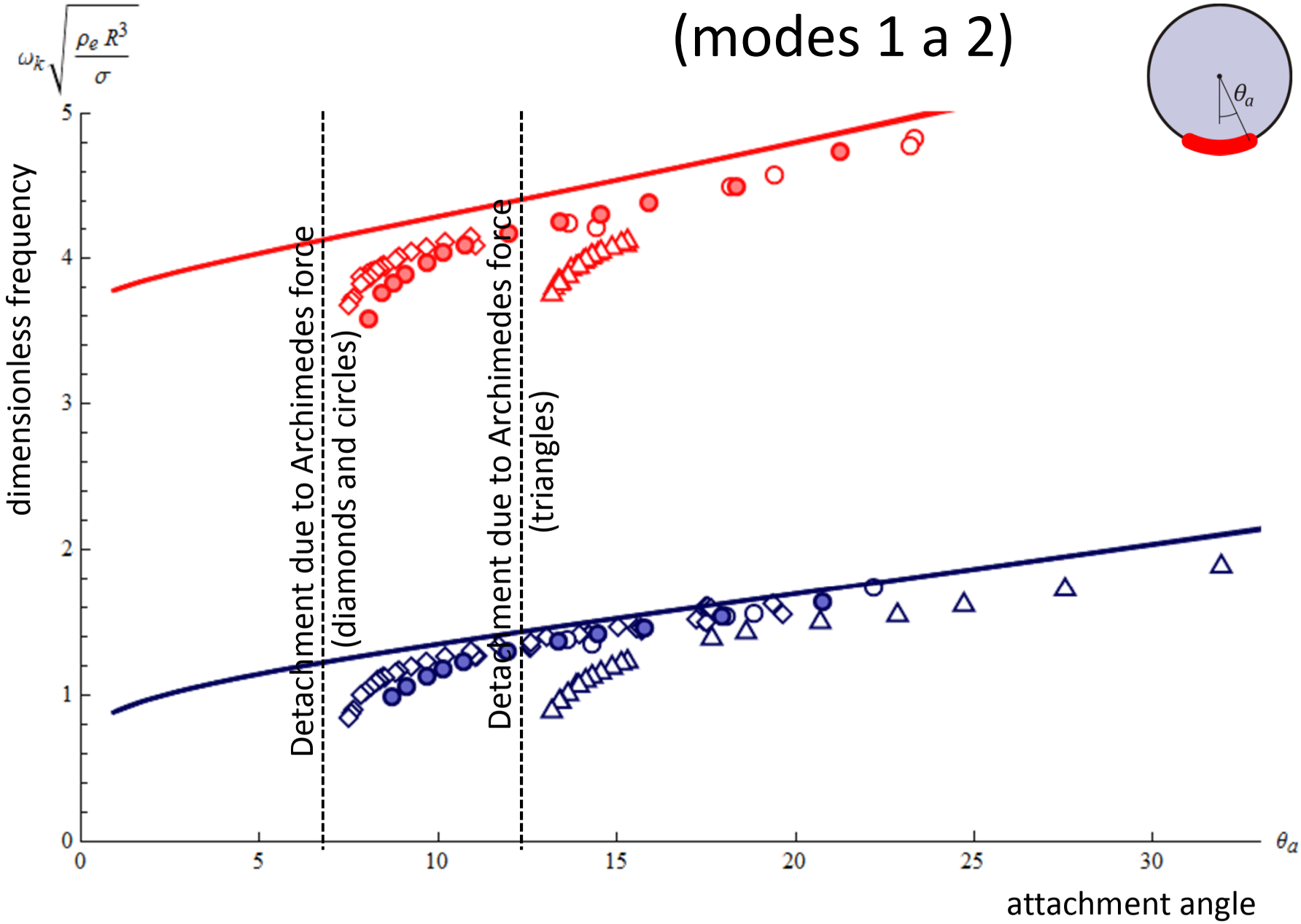
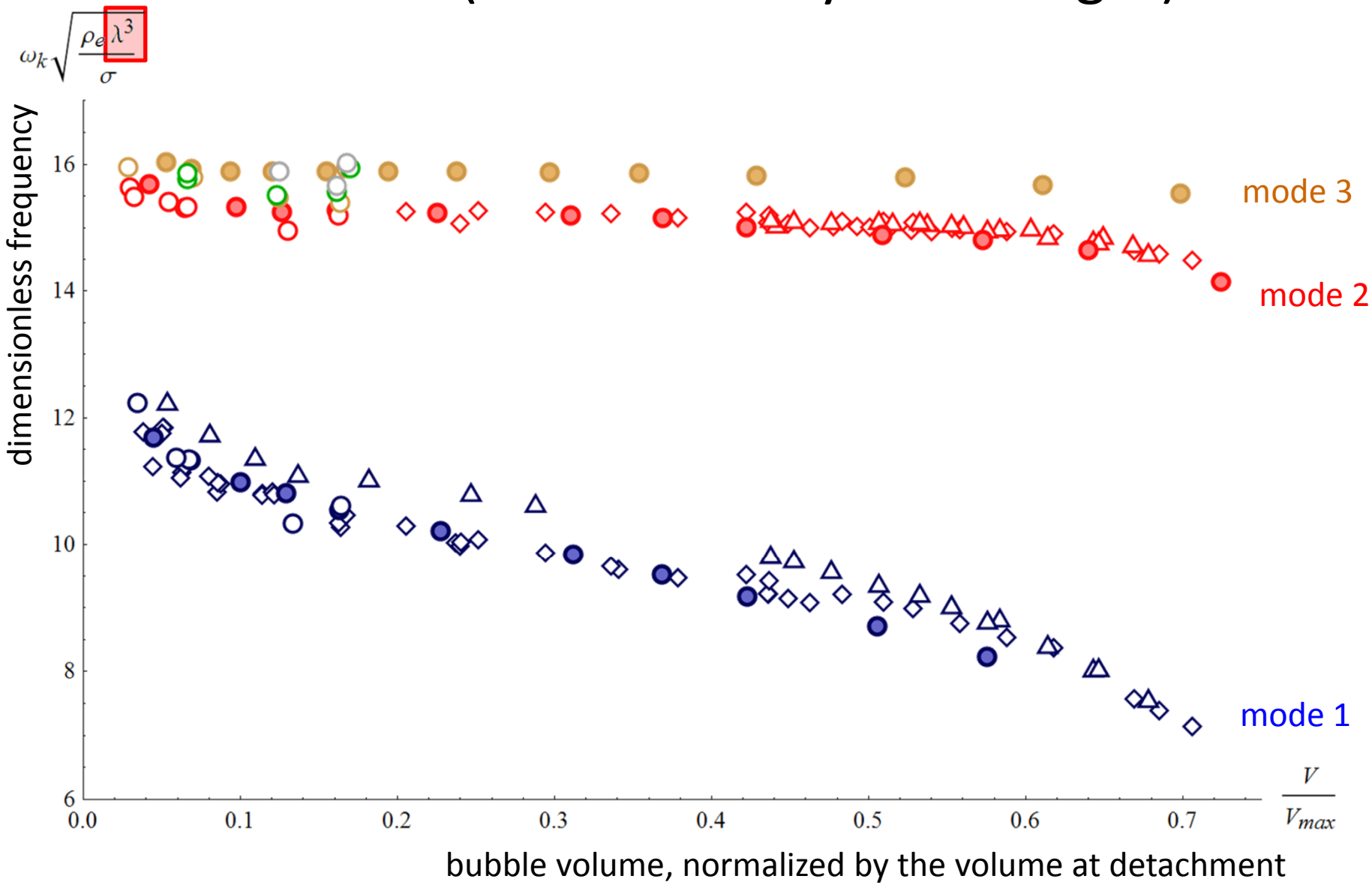


Pure liquid: eigenmode frequency (modes 1 a 2)

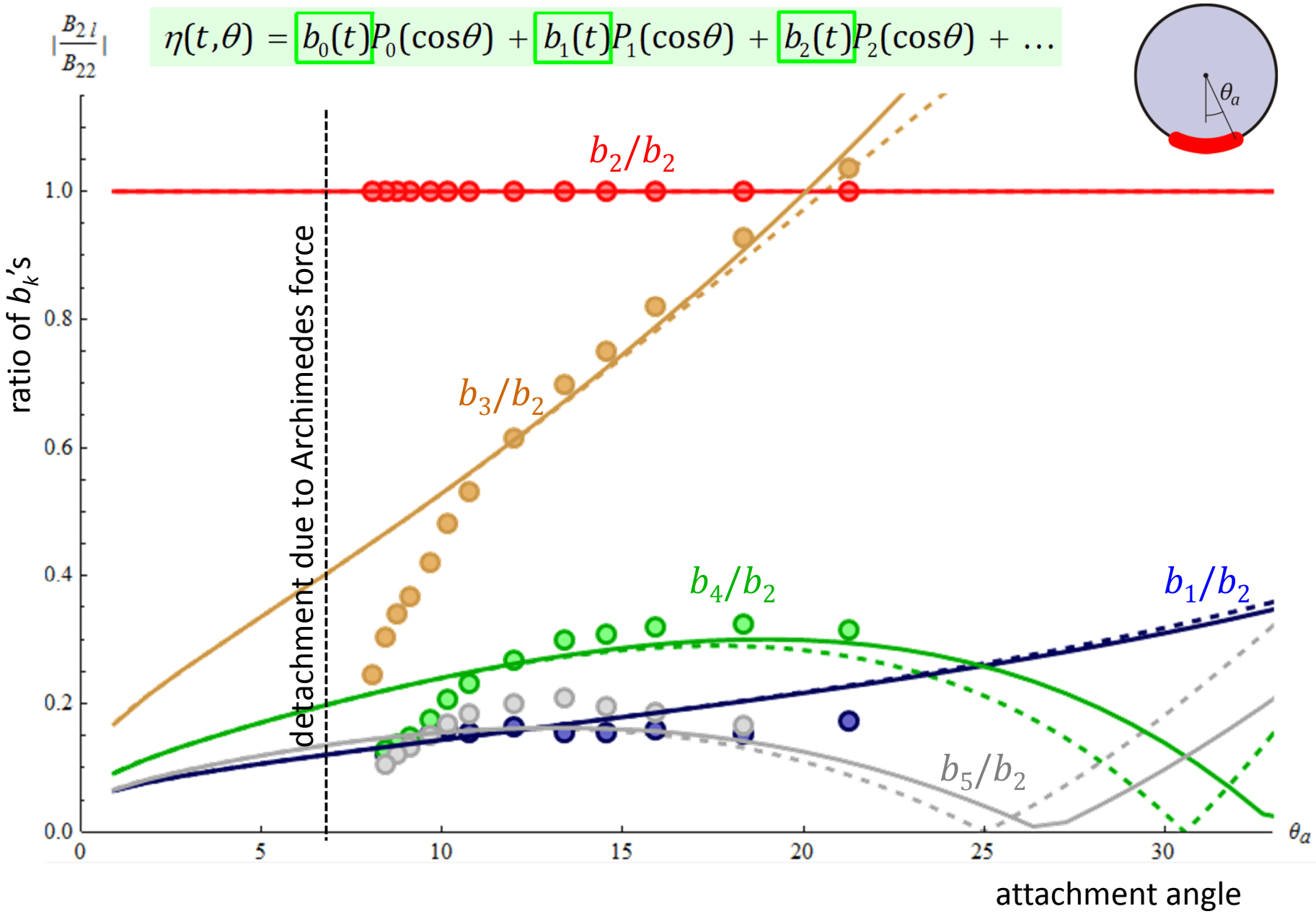


Pure liquid: eigenmode frequency (normalized by wavelength)

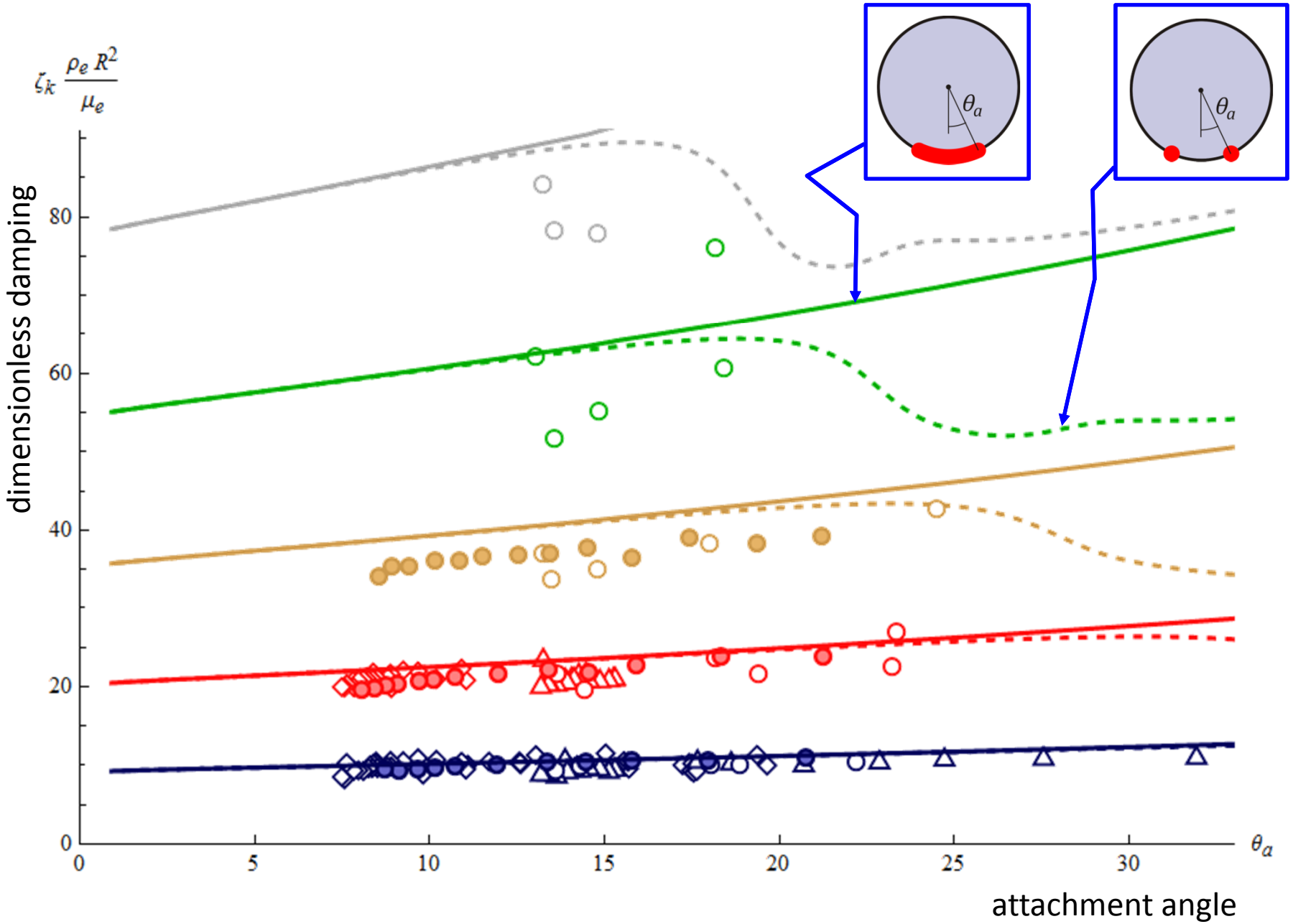


Pure liquid: eigenmode shape (mode2)

$$\eta(t,\theta) = \boxed{b_0(t)}P_0(\cos\theta) + \boxed{b_1(t)}P_1(\cos\theta) + \boxed{b_2(t)}P_2(\cos\theta) + \ldots$$



Pure liquid: eigenmode damping rate



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submitted to *Int J Multiphase Flow*

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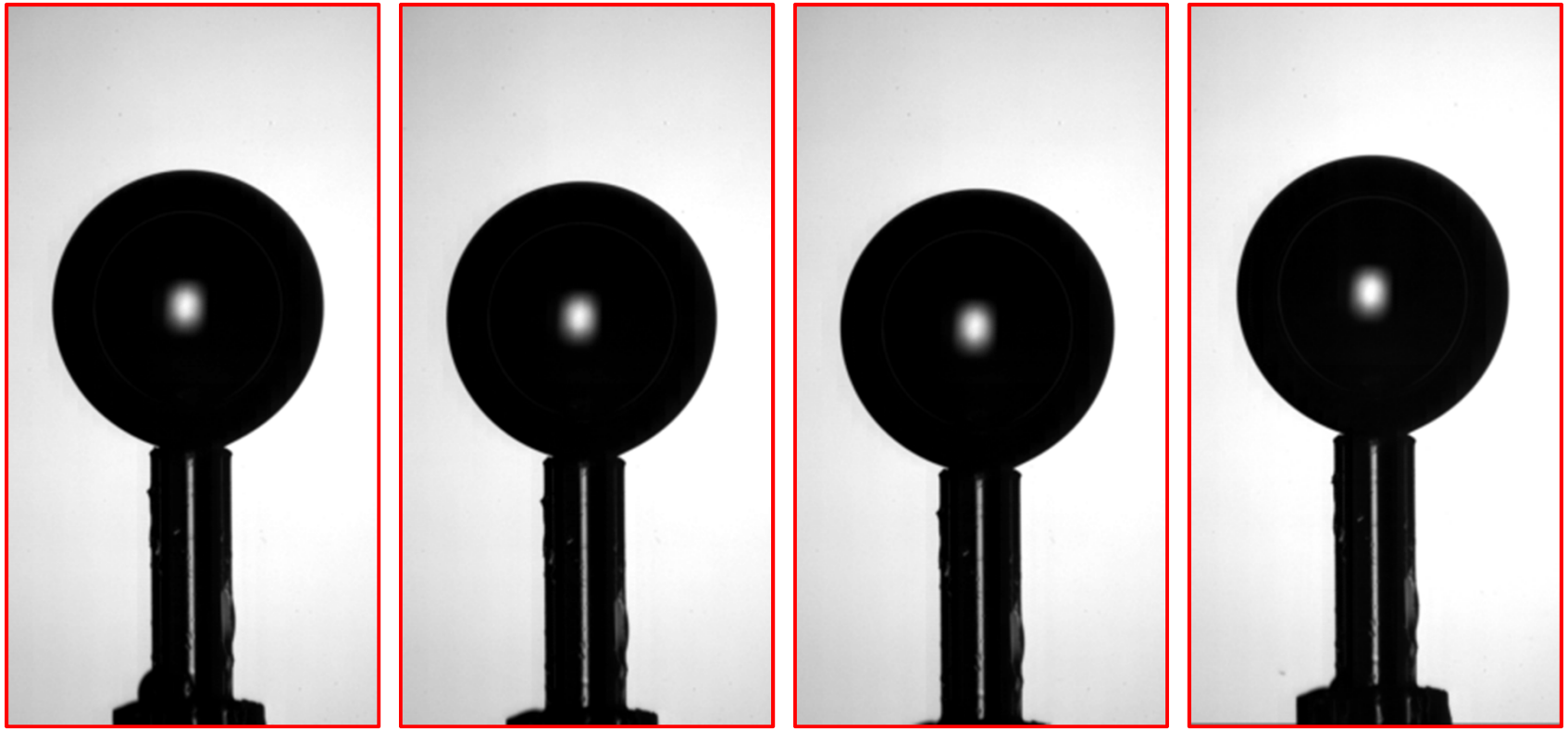
Oscillations in surfactant solutions

Bubble oscillations are suppressed, if a surfactant is present. Frequency is modified.

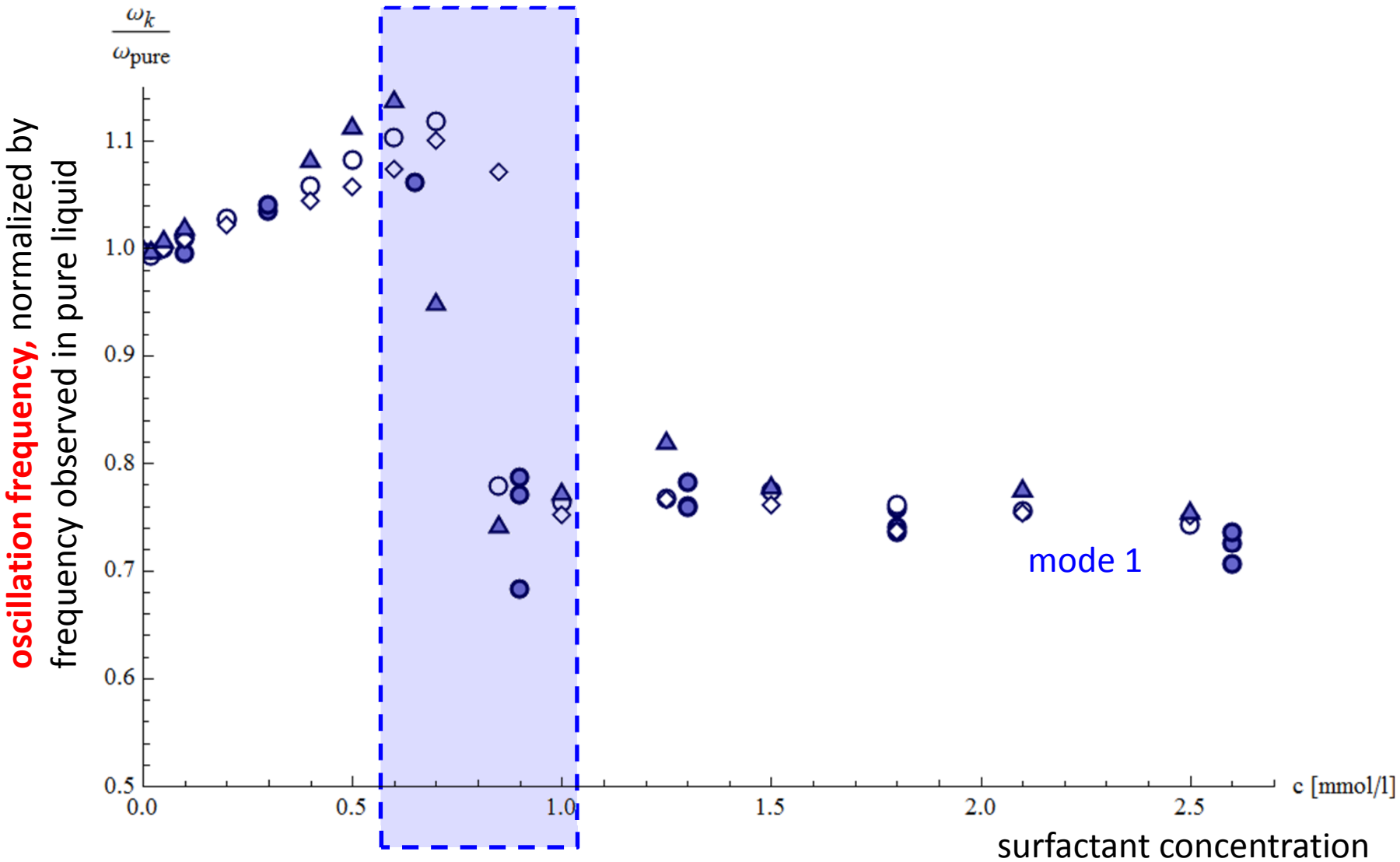
bubble size: 1.18 – 1.20 mm



pure water	water + α-terpineol	water + α-terpineol	water + α-terpineol
	c = 0.4 mmol/l (62 mg/l)	c = 0.7 mmol/l (108 mg/l)	c = 2.1 mmol/l (323 mg/l)
σ = 72 mN/m	σ = 67 mN/m	σ = 64 mN/m	σ = 56 mN/m

Effect of surfactants: oscillation frequency

results for α-terpineol



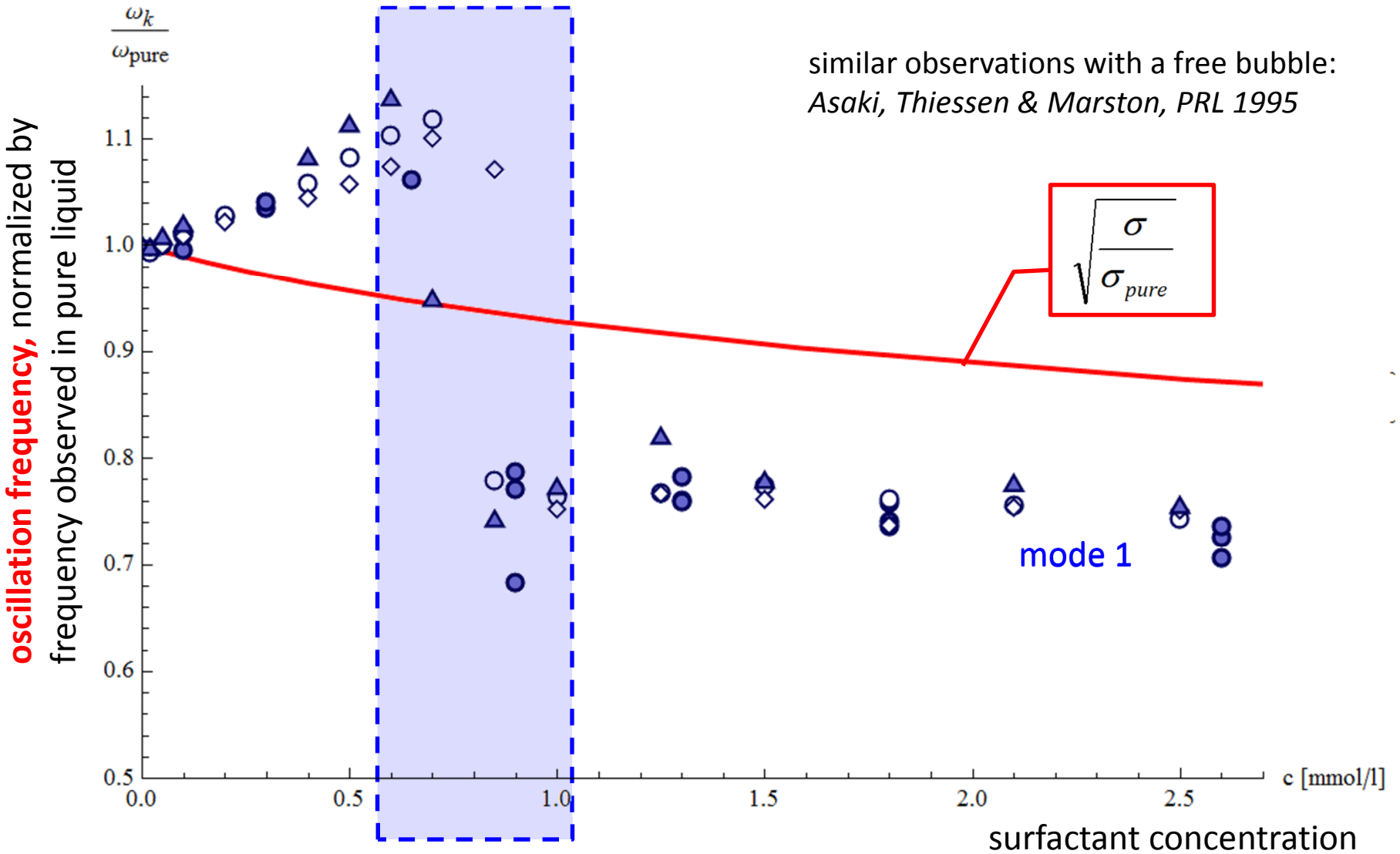
oscillation frequency, normalized by frequency observed in pure liquid

surfactant concentration

Effect of surfactants: oscillation frequency

results for α-terpineol

similar observations with a free bubble:
Asaki, Thiessen & Marston, PRL 1995



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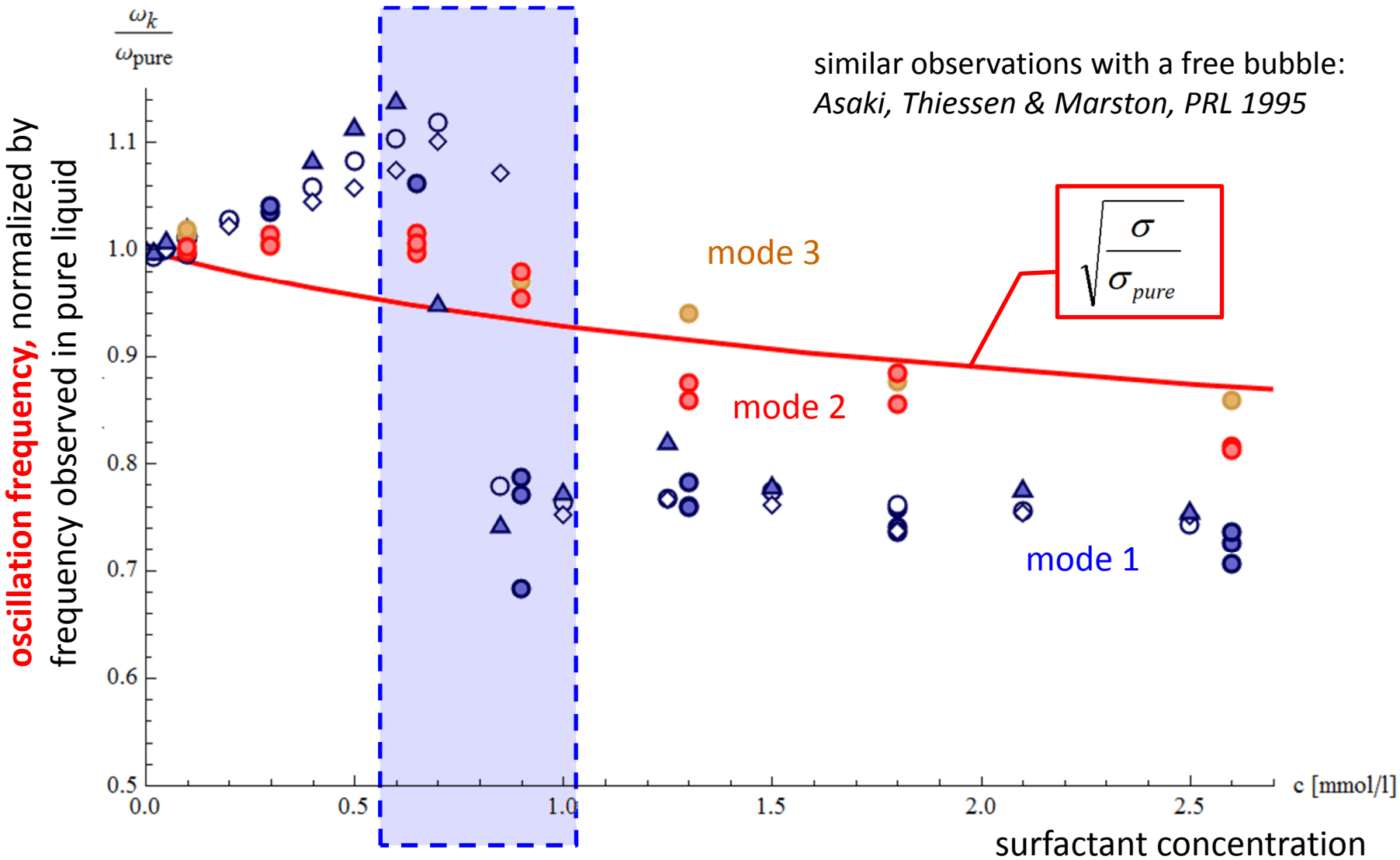

Effect of surfactants: oscillation damping

decay time = 1/damping

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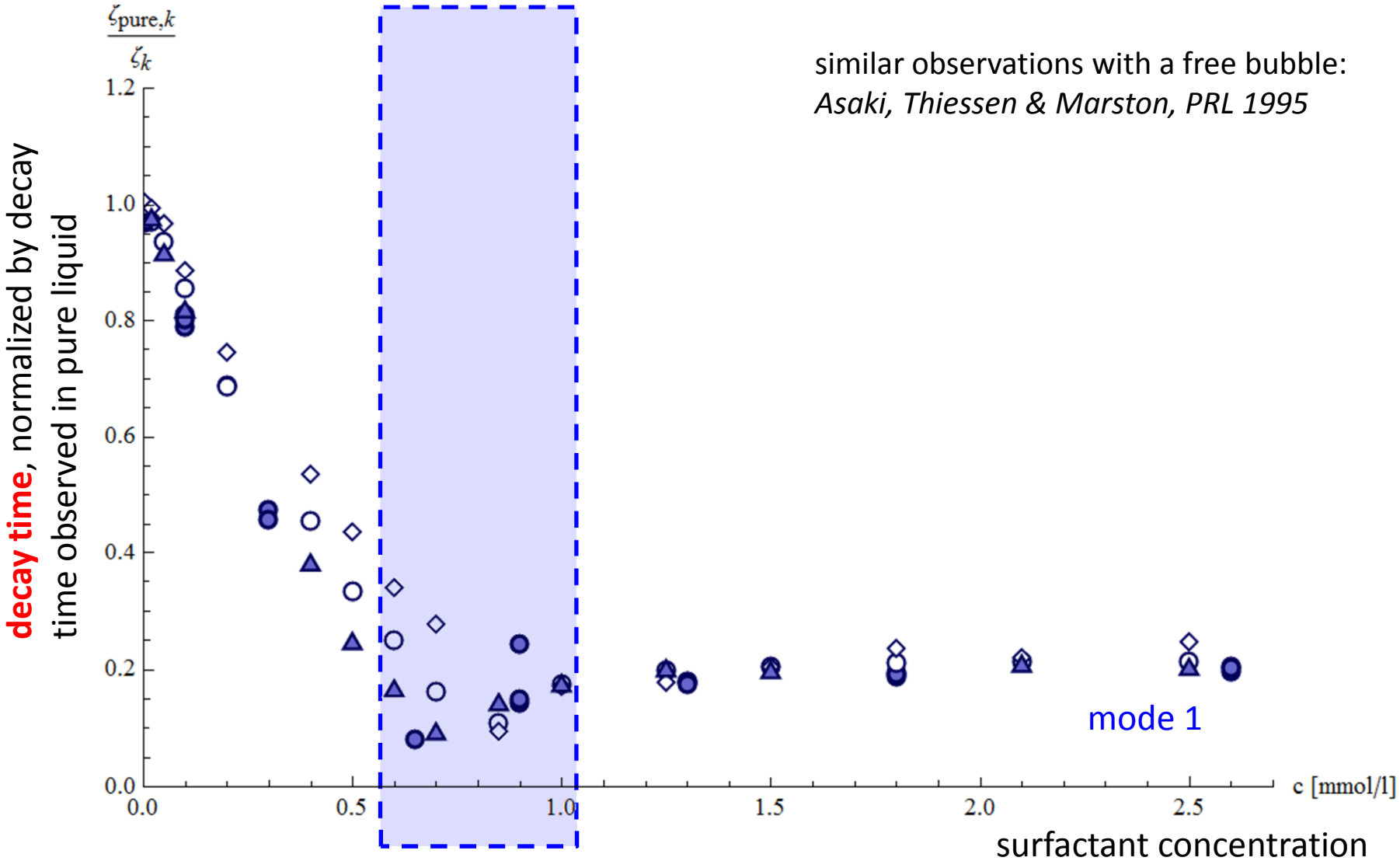

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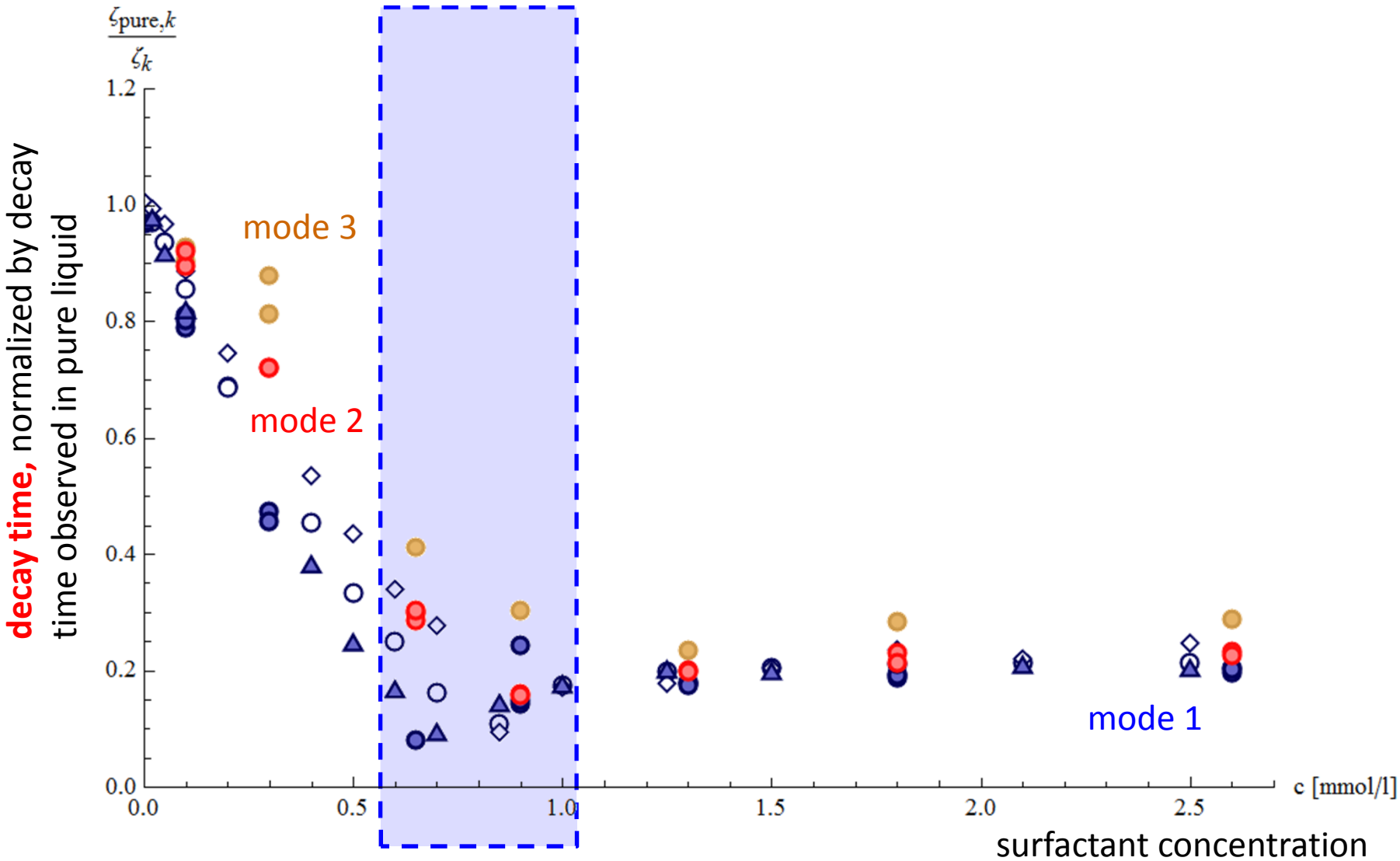

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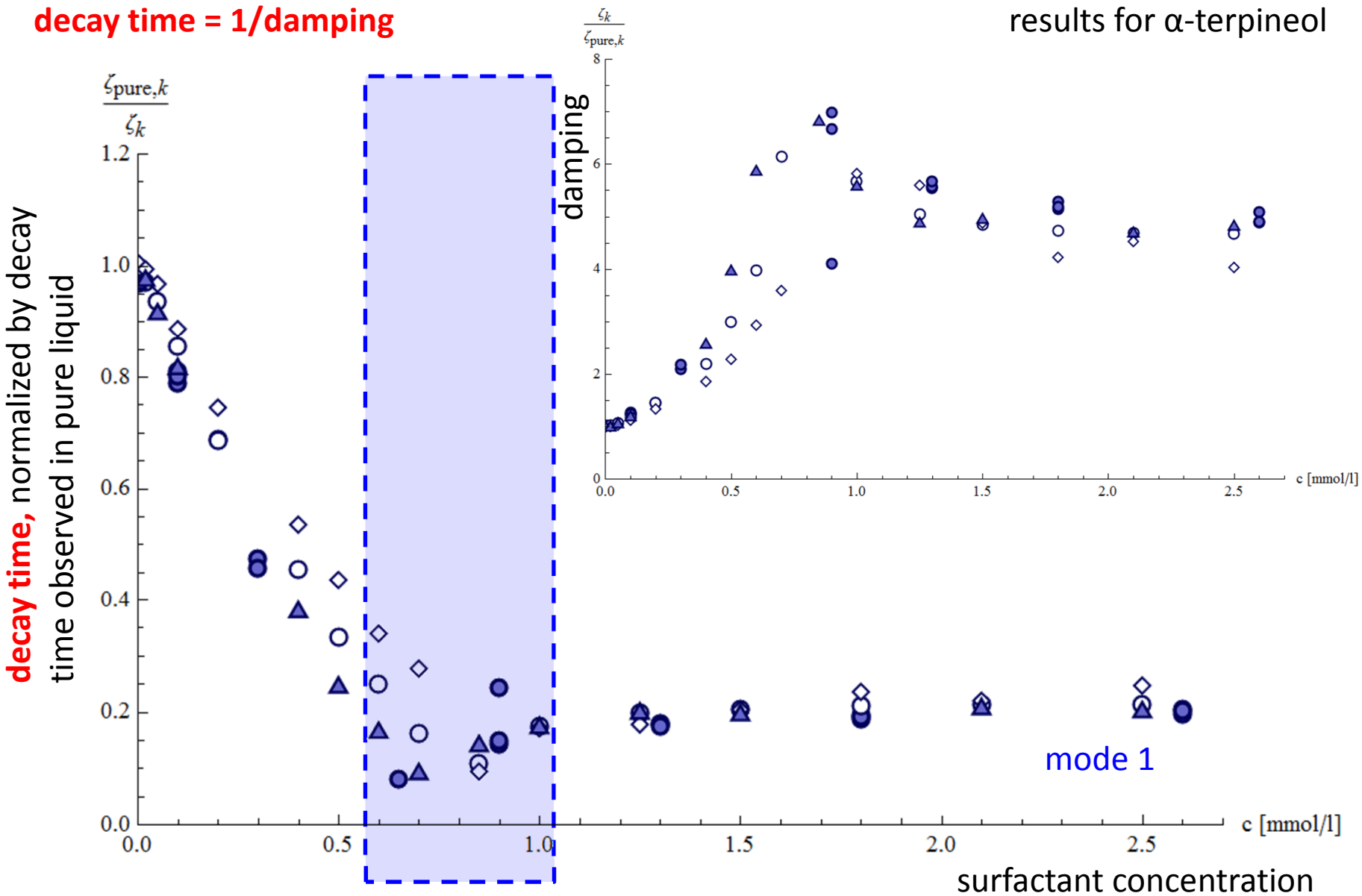
decay time = 1/damping

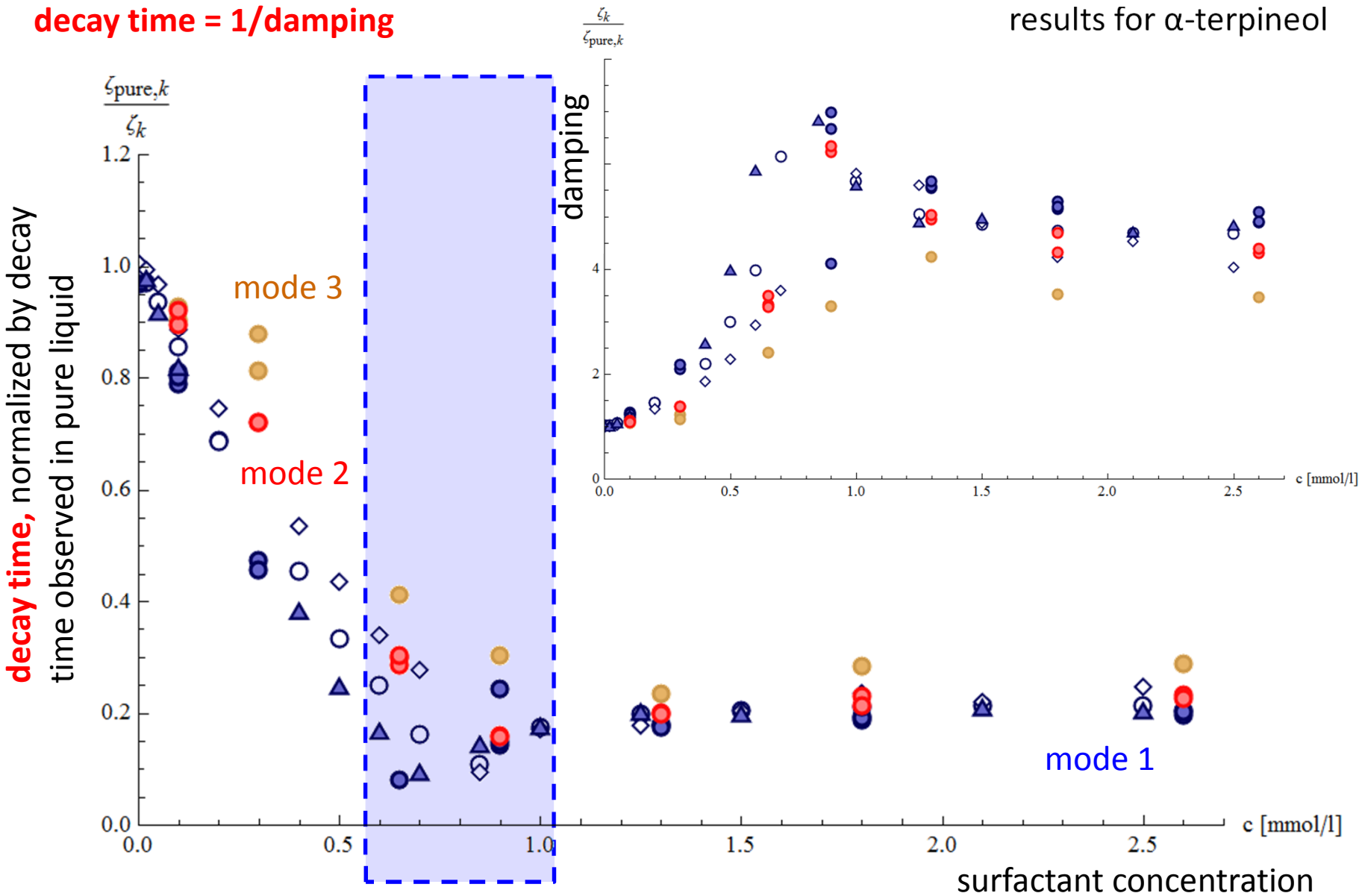
results for α-terpineol



Effect of surfactants: oscillation damping

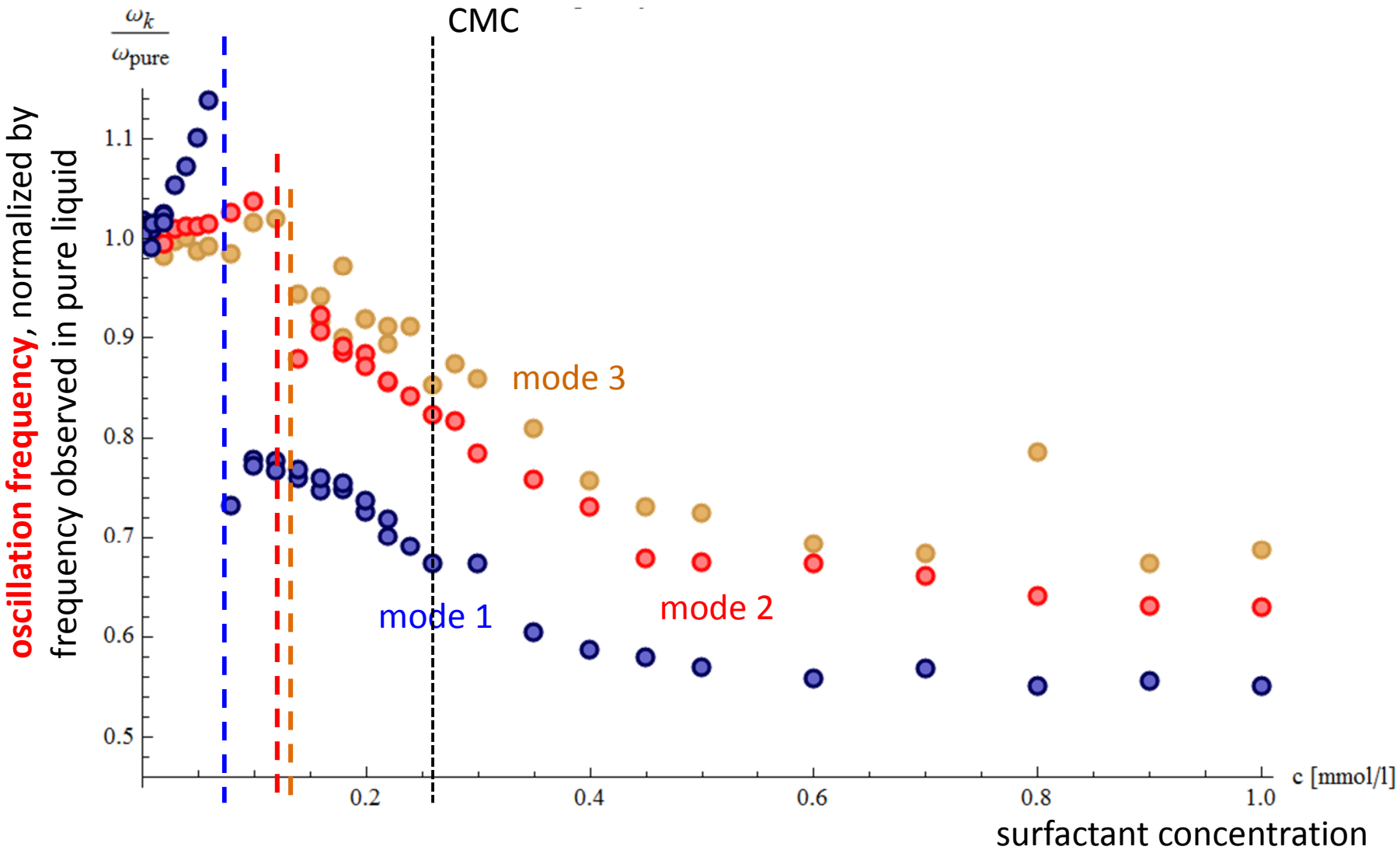
decay time = 1/damping

results for α-terpineol



Effect of surfactants: oscillation frequency

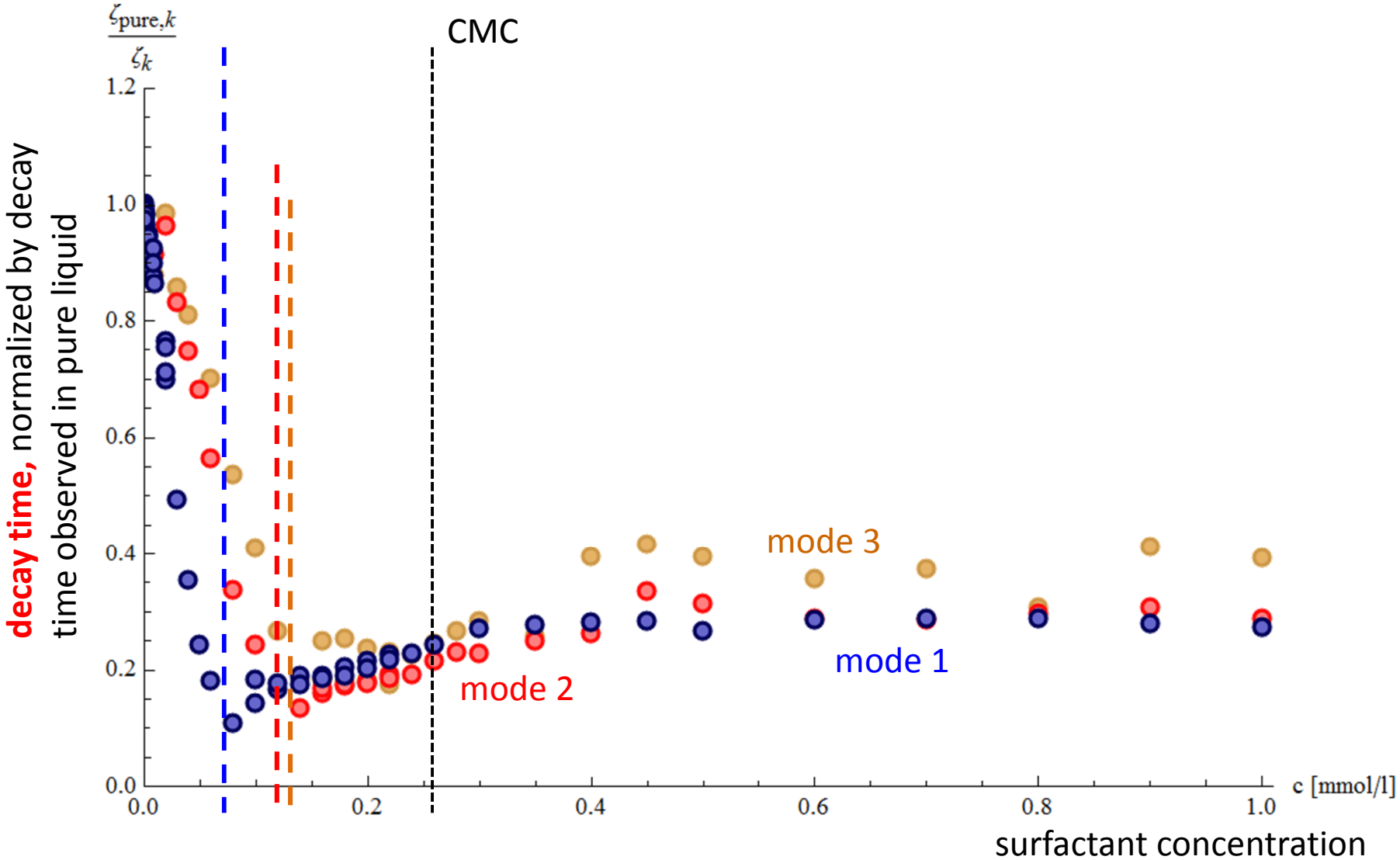
results for Triton X-100



Effect of surfactants: oscillation damping

decay time = 1/damping

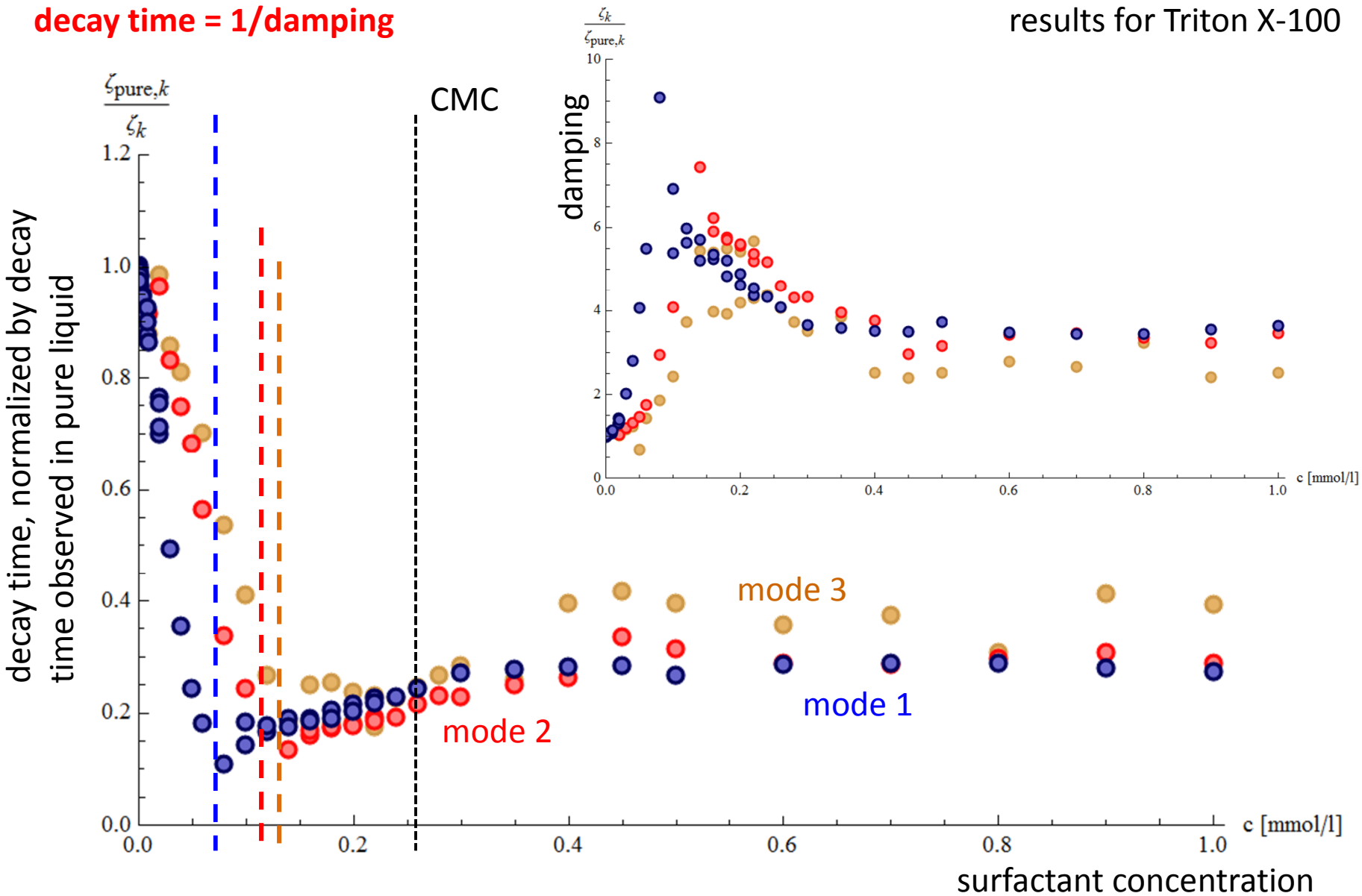
results for Triton X-100



Effect of surfactants: oscillation damping

decay time = 1/damping

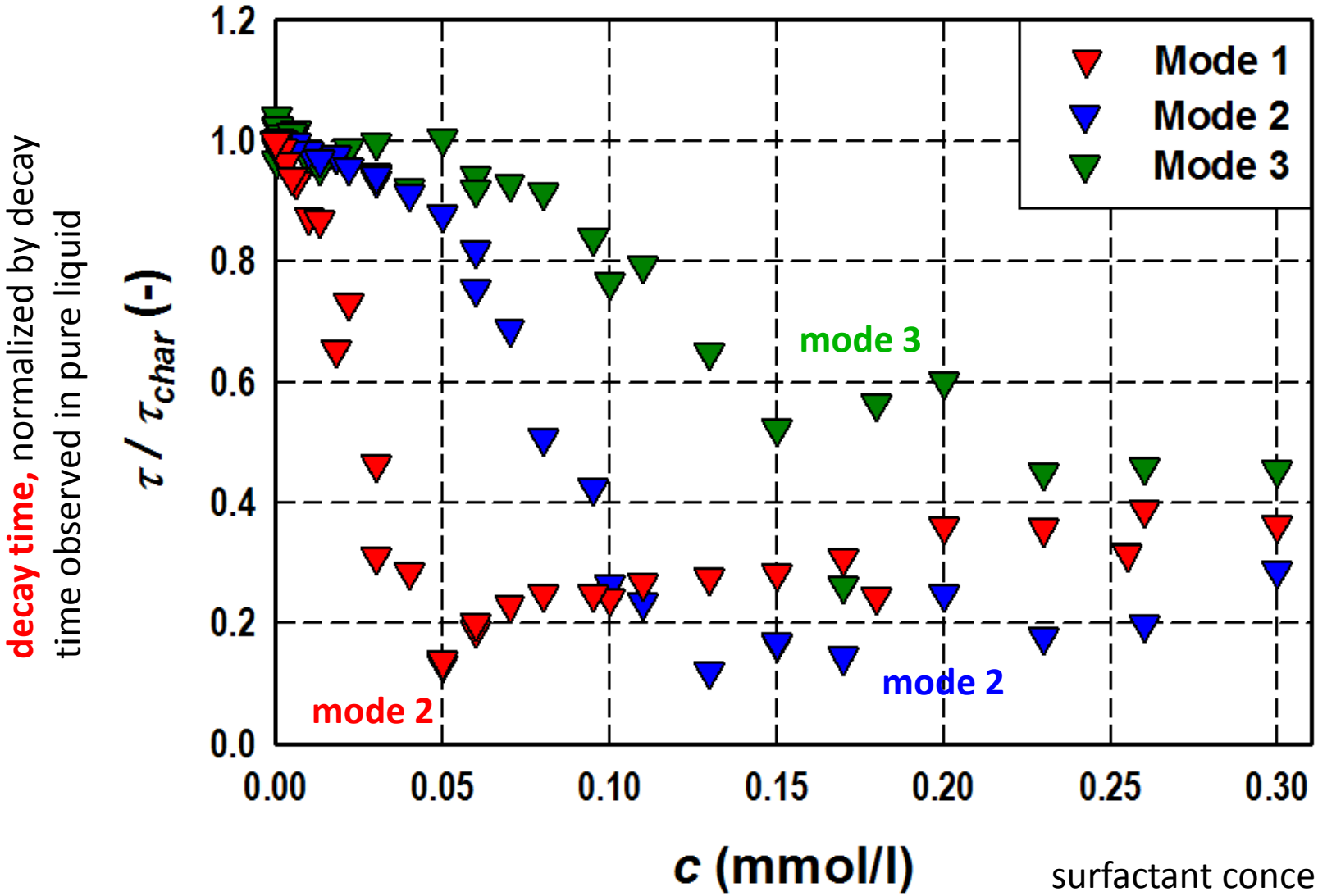
results for Triton X-100



Effect of surfactants: oscillation damping

decay time = 1/damping

results for Triton X-45



surfactant concentration

Outline

1. shape oscillations of an attached bubble
   - **inviscid analysis**
   - in pure liquids, no gravity effects

2. shape oscillations of attached bubble
   - **experiments**
   - in pure liquid, **with gravity effects**

3. shape oscillations of attached bubble
   - experiments
   - **in surfactant solutions**, with gravity effects

4. **bubble rebound from solids**
   - experiments, in surfactant solutions
   - **correlation of rebound coefficient**

5. **shape oscillations of freely-floating bubbles**
   - experiments
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Bubble rebound from a solid surface

Bubble rebound from a solid surface is suppressed, if a surfactant is present.

Left:
1 mm bubble
pure water
σ = 72 mN/m

Right:
1 mm bubble
water + surfactant
σ = 62 mN/m

Bubble rebound from a solid surface

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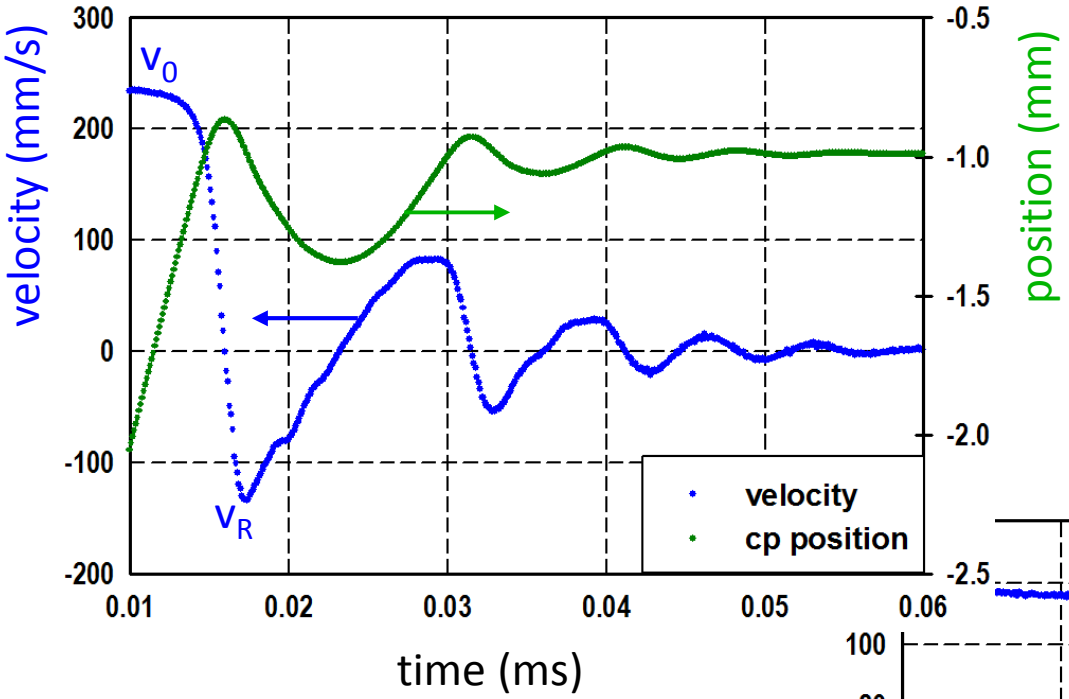
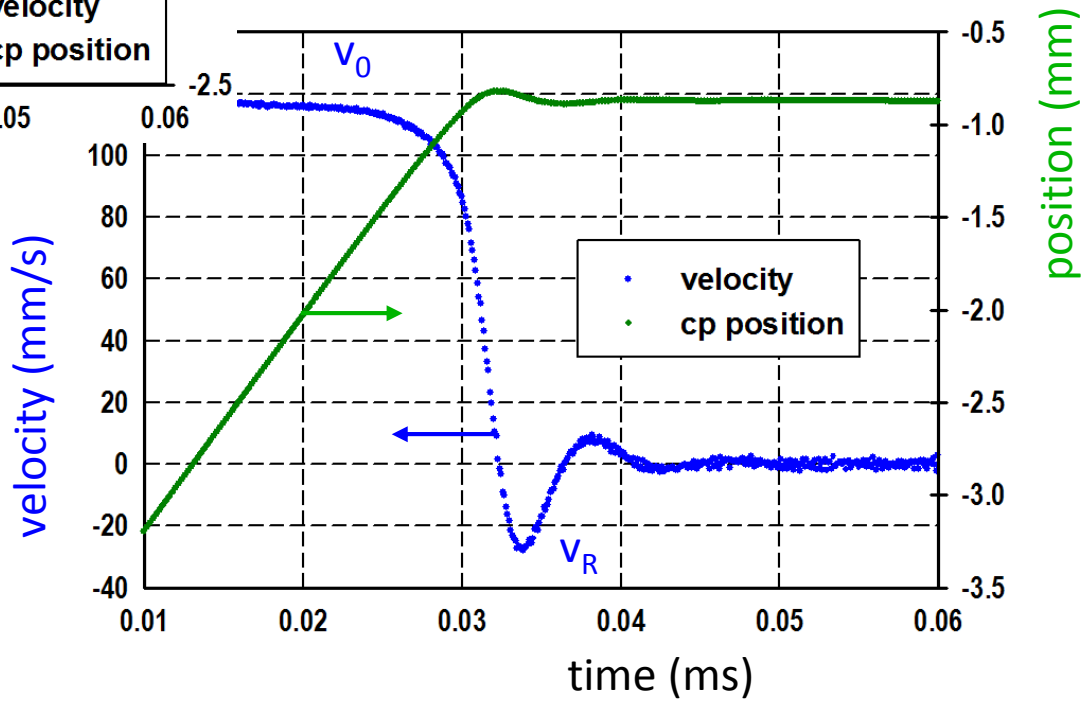


restitution coefficient: v_R/v_0



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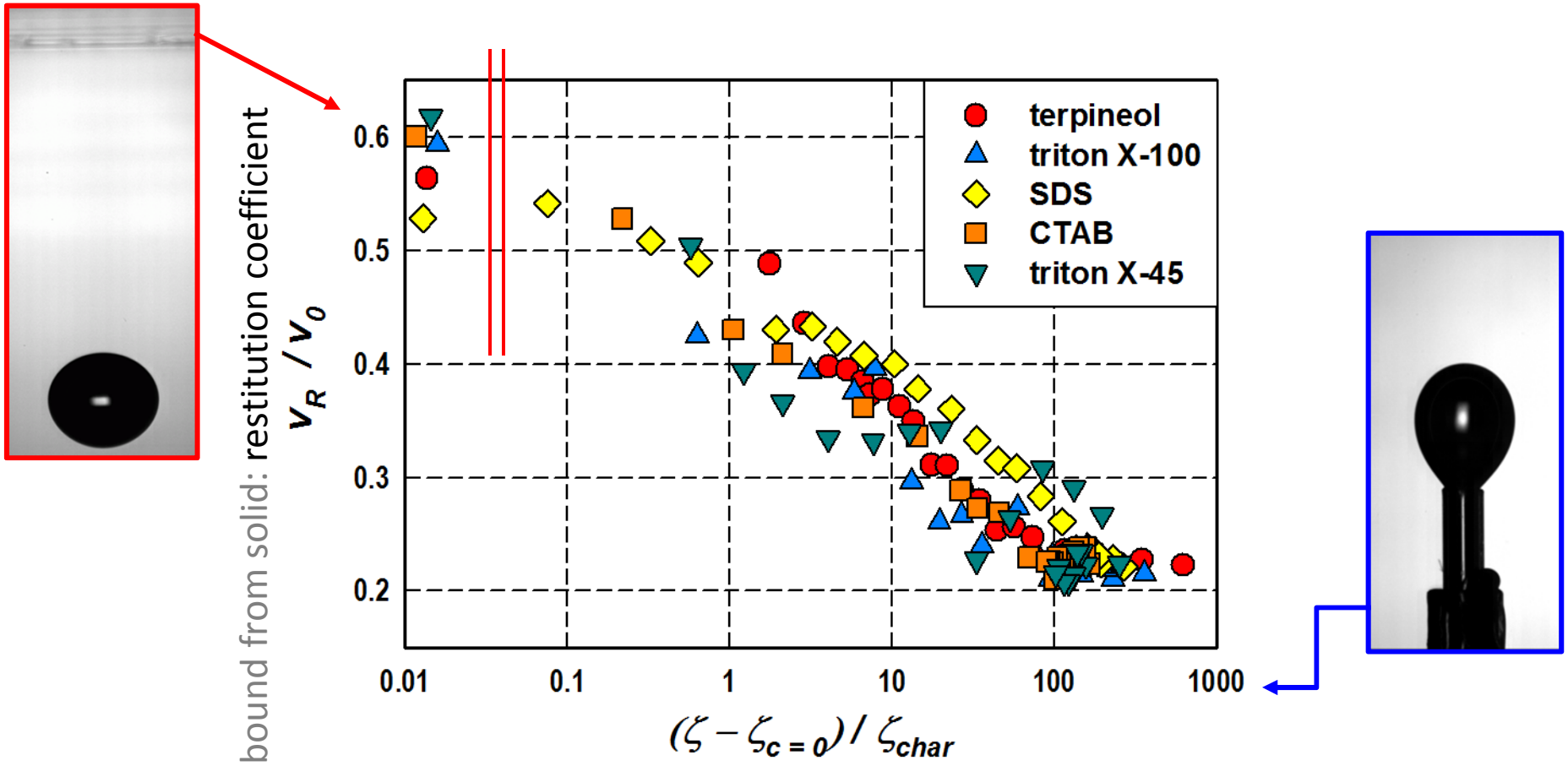

Rebound from solid: restitution coefficient

Shape oscillations: Increase of the damping rate due to surfactant

The restitution coefficient correlates with the change of damping rate of oscillations.

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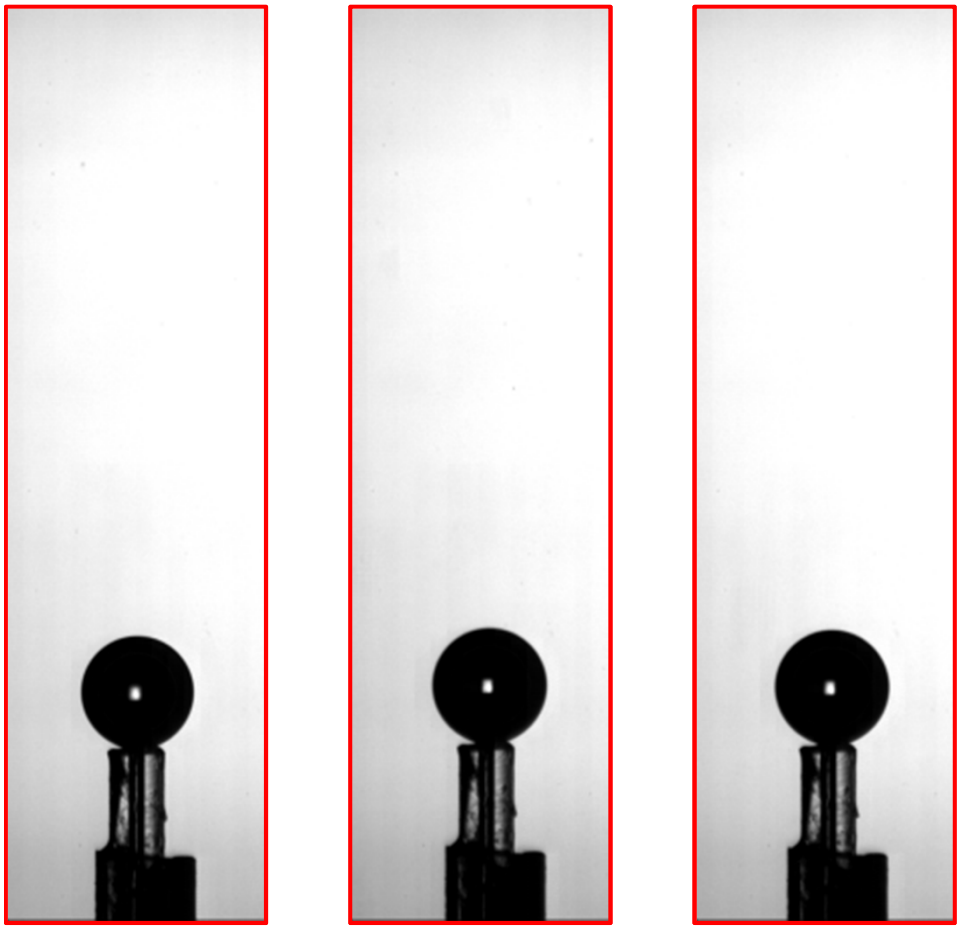
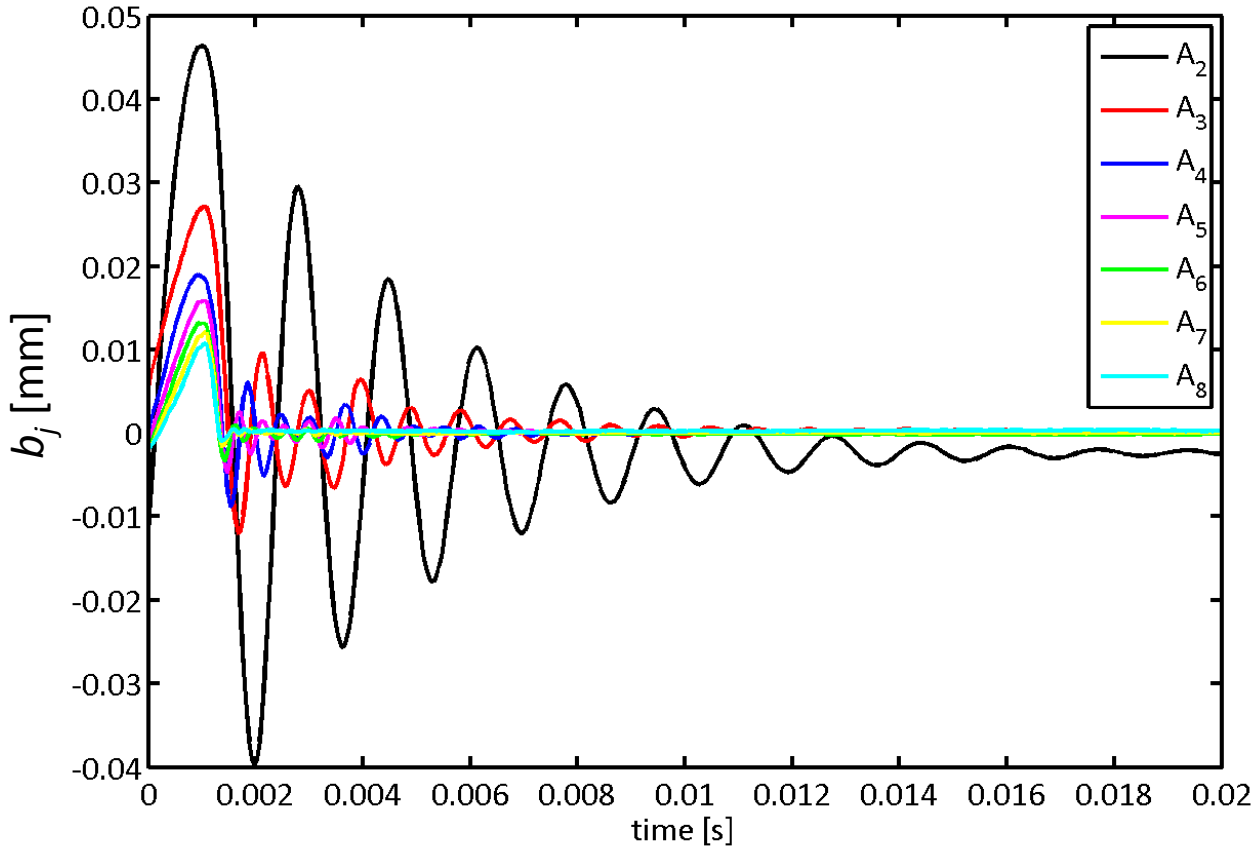
Surfactants & freely-rising bubble

Bubble oscillations are suppressed, if a surfactant is present.



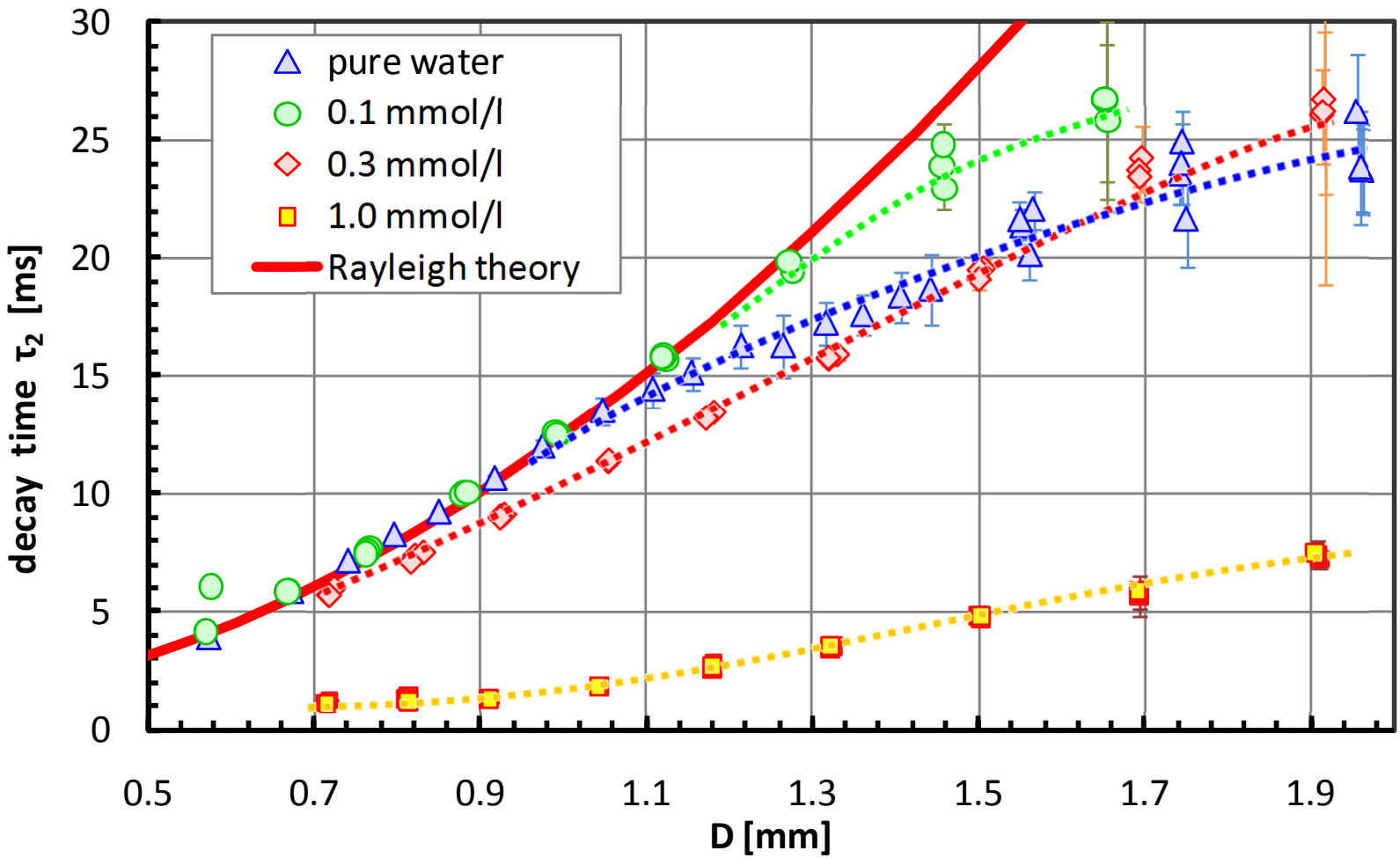
pure water	water + terpineol	water + terpineol
σ = 72 mN/m	σ = 64 mN/m	σ = 61 mN/m

Surfactants & freely-rising bubble



Surfactants & freely-rising bubble

- A decrease of frequency and maximum damping is again observed
- **Large bubbles:** The decay time can initially increase with the addition of the surfactants



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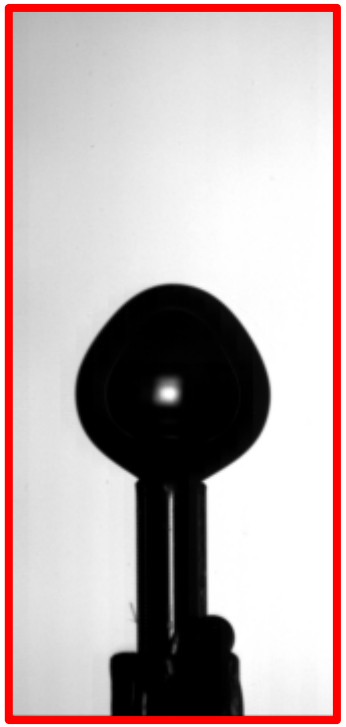
THAT'S ALL. Let us summarize.

Summary



Shape oscillations in pure liquid:

- inviscid analysis that allows to take in account any constrains, including support motion and volume variations
- deformation by gravity leads to a decrease of eigenmode frequencies
- **perspective**: analysis for bubbles/drops deformed by gravity

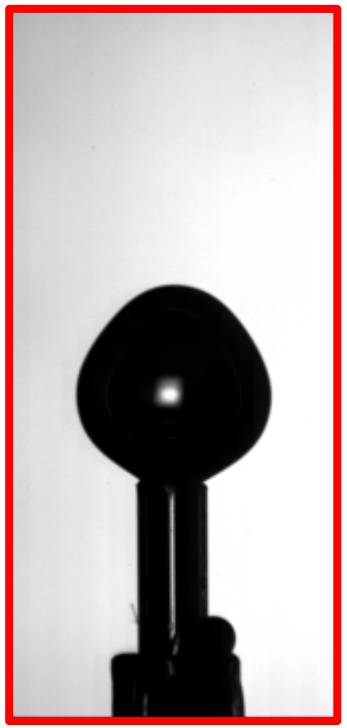
Shape oscillations in surfactant solution:

- frequency initially increases, as a surfactant is added, then frequency drops down
- maximum damping occurs at the same concentration as the frequency drop
- more complex behavior for rising bubbles

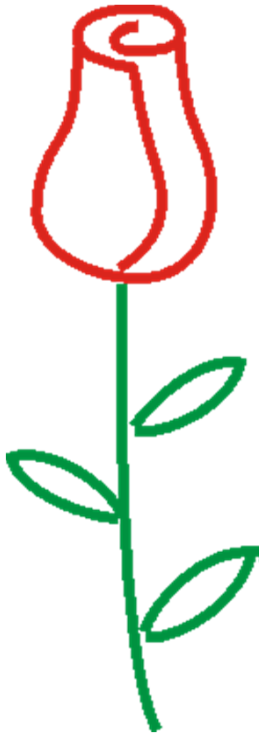
Rebound of bubbles from a solid surface:

- surfactant effect on the restitution coefficient is well correlated by the increase of damping rate of bubble oscillations

Acknowledgements



- **audience for attention**

- **Frédéric Risso, Nicolas Abi Chebel (IMFT Toulouse) and unknown Referee for interesting discussions**

- **research supported by Czech Science Foundation (P101/11/0806)**